Simulations of neutron noise in the research reactor AKR-2: comparison between a discrete ordinates and a diffusion-based method

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ABSTRACT

A diffusion-based and a discrete ordinates method are used to simulate a neutron noise experiment in the research reactor AKR-2 at Technische Universität Dresden, Germany. The AKR-2 reactor provides an interesting case for the comparison between the two methods because it is characterized by large heterogeneities and regions with low macroscopic neutron cross sections. For the calculations, the same spatial discretization and the same set of two-energy macroscopic neutron cross sections with isotropic scattering are used. Significant discrepancies between the diffusion-based and discrete ordinates methods are found in regions of the systems where the diffusion approximation is expected to be inaccurate in reproducing characteristics of the static neutron flux and neutron noise.

KEYWORDS: Neutron noise, Diffusion theory, Discrete ordinates method, Research reactor

1. INTRODUCTION

In the CORTEX project [1], different solvers were developed to simulate neutron noise in neutron multiplying systems, i.e., fluctuations of the neutron flux induced by small, stationary perturbations of the macroscopic neutron cross sections. These solvers were compared, e.g., in the case of an oscillating perturbation of the cross sections of a fuel pin in a simplified fuel assembly, and it was showed that diffusion-based and higher-order transport methods may provide similar results, although discrepancies can be found close to the location of a perturbation or in case of abrupt changes in the material properties, see [2].

In the current study, comparisons between the diffusion approximation and a discrete ordinates method are further investigated for the simulation of a neutron noise experiment in the research reactor AKR-2 at Technische Universität Dresden – TUD, Germany [3]. For the purpose, the diffusion-based solver CORE

SIM+ [4] and the discrete ordinates solver NOISE-SN [5] are used. The AKR-2 reactor provides an interesting test because it is characterized by large heterogeneities and by regions with low macroscopic total cross sections, which may cause the diffusion approximation to be inaccurate.

The paper is structured as follows. In Section 2, the reactor and the experiment are described. In Section 3, the modelling is discussed. In Section 4, the CORE SIM+ and NOISE-SN results are compared. In Section 5, conclusions are drawn.

2. DESCRIPTION OF AKR-2 AND THE NEUTRON NOISE EXPERIMENT

The thermal zero-power research reactor AKR-2 includes a cylindrical core, a reflector region, a shielding region, and various empty channels, see Fig. 1 [3]. The core has a diameter of 25 cm and a height of 27.5 cm and consists of disk-shaped fuel elements with a homogeneous mixture of polyethylene and uranium oxide with 19.8% enrichment in U-235. Three control/safety rods can be vertically inserted into positions next to the fuel zone to control the neutron population. The reflector region is made of graphite and surrounds the core. A layer of paraffin (whose thickness is 15 cm) and an outer layer of heavy concrete (whose thickness is 58 cm) serve as biological shields. Between these two layers, there is an air gap. The reactor has four horizontal and two vertical empty, air-filled cylindrical channels for instrumentation and experiments. These channels may introduce challenges in the modelling and simulations because of their relatively large dimensions and low neutron cross sections.

In the experiment simulated in this work, neutron noise is induced by a neutron absorber of variable strength. The noise source is a thin foil of cadmium (its length is 15 cm, its width is 2 cm, and its thickness is 0.02 cm) that rotates at a frequency of 1 Hz, along a circumference with radius of 2.98 cm, in the middle of channel 3-4 (the channel is shown in Fig. 1 and its radius is 3.5 cm). Since cadmium is a thermal neutron absorber, the rotation of the foil perturbs locally the thermal neutron capture cross section. Given the size of the foil, such a perturbation is small.

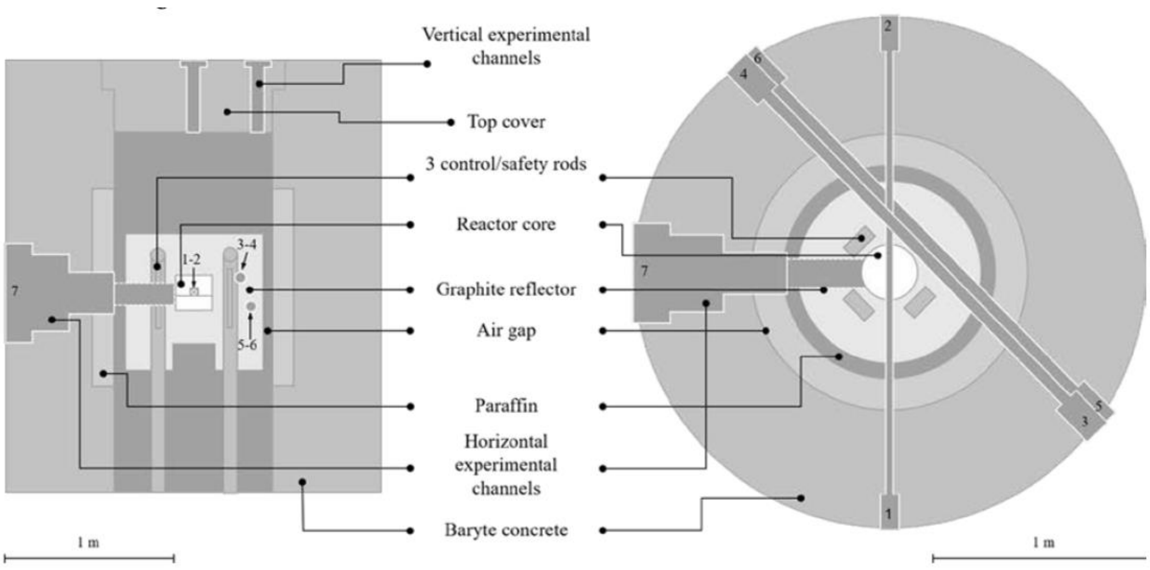

Figure 1. Schematics of the AKR-2 reactor (courtesy of TUD)

3. MODELLING

Two neutron noise solvers are used for the simulations, namely CORE SIM+ [4] and NOISE-SN [5]. The neutron noise equation used in the solvers is introduced. Then the modelling of the AKR-2 reactor and of the neutron noise experiment is described. Details of the numerical solution are also provided.

3.1. Frequency-domain neutron noise transport equation

The solvers CORE SIM+ and NOISE-SN are based on the following frequency-domain neutron noise equation (where the notation is standard):

$$\left[\hat{\Omega}\cdot\nabla+\Sigma_{t,g,0}(\vec{r})+\frac{i\omega}{v_g}\right]\delta\psi_g(\vec{r},\hat{\Omega},\omega)=\frac{1}{4\pi}\sum_{g'}\Sigma_{s,g'\to g,0}(\vec{r})\delta\phi_{g'}(\vec{r},\omega)+$$

$$\frac{1}{4\pi k_{eff}}\left[\chi_{p,g}(\vec{r})\left(1-\sum_{q}\beta_q(\vec{r})\right)+\sum_{q}\chi_{q,g}(\vec{r})\frac{\lambda_q\beta_q(\vec{r})}{i\omega+\lambda_q}\right]\sum_{g'}\nu\Sigma_{f,g',0}(\vec{r})\delta\phi_{g'}(\vec{r},\omega)+S_g(\vec{r},\hat{\Omega},\omega)\tag{1}$$

In Eq. (1), the neutron noise source $S_g(\vec{r},\hat{\Omega},\omega)$ is expressed as:

$$S_g(\vec{r},\hat{\Omega},\omega)=-\delta\Sigma_{t,g}(\vec{r},\omega)\psi_{g,0}(\vec{r},\hat{\Omega})+\frac{1}{4\pi}\sum_{g'}\delta\Sigma_{s,g'\to g}(\vec{r},\omega)\phi_{g',0}(\vec{r})+$$

$$\frac{1}{4\pi k_{eff}}\left[\chi_{p,g}(\vec{r})\left(1-\sum_{q}\beta_q(\vec{r})\right)+\sum_{q}\chi_{q,g}(\vec{r})\frac{\lambda_q\beta_q(\vec{r})}{i\omega+\lambda_q}\right]\sum_{g'}\nu\delta\Sigma_{f,g'}(\vec{r},\omega)\phi_{g',0}(\vec{r})\tag{2}$$

The unknown quantities are the angular neutron noise $\delta\psi_g$ and the scalar neutron noise $\delta\phi_g$, and take complex values. The parameter ω is the angular frequency of the perturbation and i is the imaginary unit number. The scattering is assumed isotropic. The derivation of Eqs. (1)-(2) can be found, e.g., in [5].

For the solution, CORE SIM+ uses the diffusion approximation, while NOISE-SN the discrete ordinates method. In both cases, first, the determination of the static scalar neutron flux $\phi_{g',0}(\vec{r})$ (which is normalized before its use in the neutron noise calculations) and the effective multiplication factor k_{eff} (which is assumed constant in the neutron noise calculations) are needed.

3.2. Spatial discretization of AKR-2

The spatial discretization of AKR-2 used for the CORE SIM+ and NOISE-SN simulations is shown in Fig. 2. In the axial direction, only a part of the reactor is modelled to avoid unnecessary computational effort, and includes a bottom reflector, the core, and a top reflector. In the radial direction the model consists of the full reactor, i.e., the core at the center, the reflector, the air gap, the paraffine region, and the shielding concrete region in the periphery. The spatial grid is such that finer computational nodes are used for the central region including the core and the reflector. Each region has its own set of 2-energy group, homogenized macroscopic neutron cross sections, assuming the scattering to be isotropic. These macroscopic cross sections are generated using a model provided by TUD and the Monte Carlo code Serpent [6] together with the ENDF/B-VII nuclear data libraries.

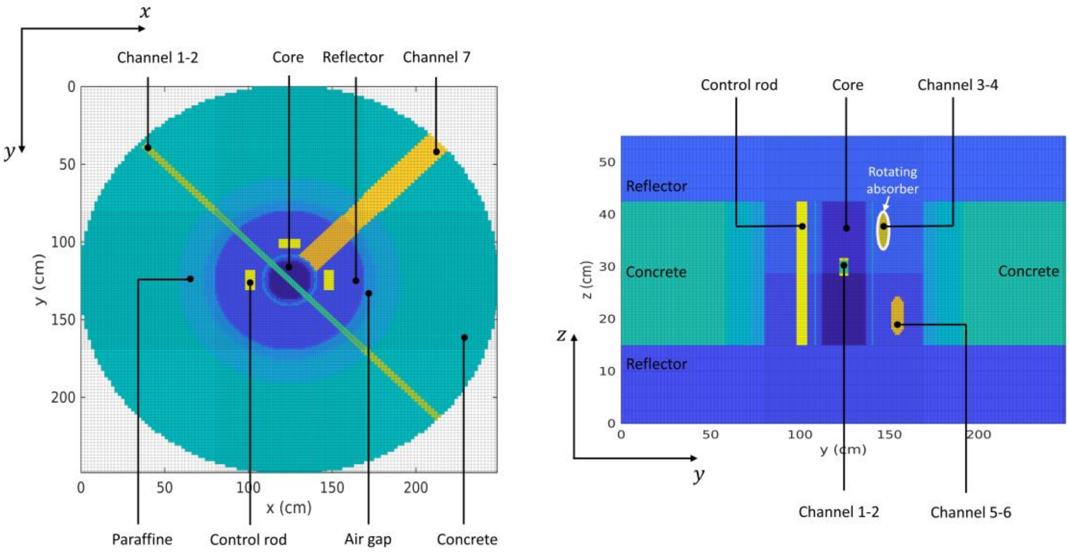

Figure 2. Spatial discretization of AKR-2 used for the calculations

3.3. Modeling of the rotating neutron absorber in AKR-2

The absorber of variable strength is modeled as proposed in [7]. Fluctuations of the macroscopic thermal neutron capture cross section are specified in the computational nodes of Channel 3-4 that are located next to the radial boundary and are crossed by the absorber in its rotation, see right-hand picture in Fig. 3. The perturbation in each of these computational nodes is given by:

$$\delta\Sigma_{c,2}(\theta) = \Sigma_{c,2,Cd}[\sin(\omega t) + i\cos(\omega t)]_{t=a}^{t=b}, \tag{3}$$

The parameter ω is the rotation frequency of the absorber, $\Sigma_{c,2,Cd}$ is the macroscopic thermal neutron capture cross section of the cadmium absorber. The parameter a and b are expressed as:

$$a = \frac{R\theta}{v} - \frac{T}{2}, \tag{4}$$

$$b = \frac{R\theta}{v} + t_d - \frac{T}{2}, \tag{5}$$

The quantity T is the rotation period of the absorber, v is the velocity of the absorber, t_d is equal to the length of the absorber divided by the velocity, θ is the rotation angle, and R is the radius of the circle followed by the absorber in the rotation.

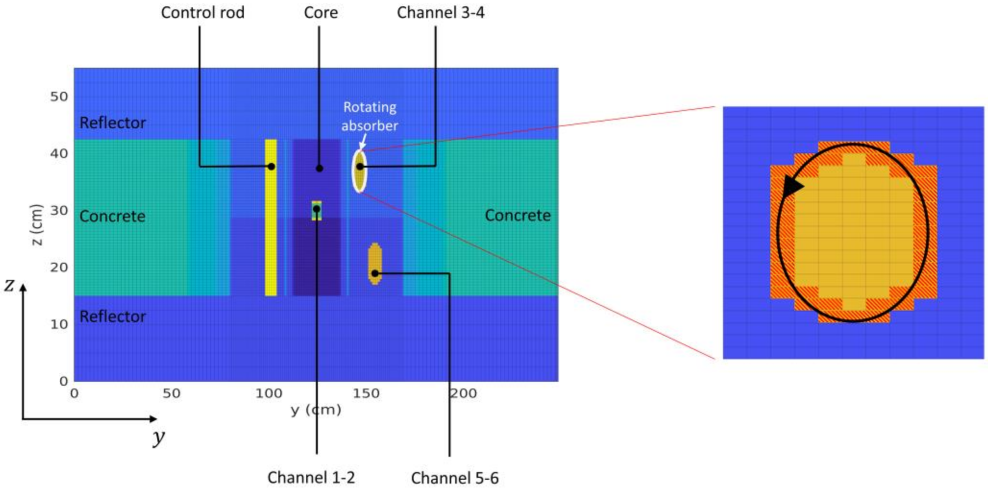

Figure 3. Spatial discretization of channel 3-4 with computational nodes perturbed by the rotation of the neutron absorber (on the right, in orange); the black circle indicates the trajectory of the absorber, and the black arrow indicates the direction of rotation.

3.3. Numerical solution

In CORE SIM+, the equations are discretized according to the finite difference scheme. The iterative linear solver used for the numerical solution of the neutron noise equations is accelerated with an ILUk preconditioner. Since the implementation of the preconditioner is limited to real-arithmetic, the original complex-valued set of equations is transformed into a real-valued equivalent problem.

In NOISE-SN, the discretization of the equations is based on a standard discrete ordinates method. The spatial differencing is obtained from the diamond finite difference method. The scalar quantities are constructed from the angular quantities using the PN-TN quadrature set. The order of discrete ordinates for the current calculations is S16. The static and neutron noise CMFD equations are discretized over the fine transport mesh since it is difficult to select an appropriate coarser mesh for the complex geometry of AKR-2.

4. RESULTS

Considering the static configuration of AKR-2, the effective multiplication factor values obtained from CORE SIM+ and NOISE-SN are 0.96344 and 1.02190, respectively. The difference is equal to -5846 pcm and the value from the diffusion method is further from unity. The Monte-Carlo Serpent model applied to generate the macroscopic cross sections estimates the effective multiplication factor equal to $1.0159 \pm 1.6e-5$.

The CORE SIM+ and NOISE-SN simulations are compared for relevant locations, in terms of the static neutron flux (normalized using the maximum value of the fast neutron flux, which occurs at the center of the core), the absolute neutron noise amplitude, and the neutron noise phase. Figure 4 shows the results at mid-elevation, along the diagonal that goes from South-West (SW) to North-East (NE) and that follows channel 7; Fig. 5 the results at mid-elevation, along the line that crosses horizontally the system in the middle; and Fig. 6 the results along the channel 5-6 (which can be used to place neutron detectors).

The normalized static neutron flux and the neutron noise amplitude have a very similar shape. This is because the system is small and behaves in a point-kinetic manner.

In the central part that includes the core, radial reflector, and paraffin zone, the differences between the normalized static neutron fluxes and between the neutron noise amplitudes calculated with CORE SIM+ and NOISE-SN are relatively small, see plots in the second and third rows of Figs. 4 and 5. Also, the solver CORE SIM+ predicts a deeper dip of the normalized thermal neutron flux and of the neutron noise amplitude at the center of the system (where the empty channel 1-2 crosses the reactor core) than NOISE-SN (not visible in the plots).

At the interface between the core and channel 7, CORE SIM+ predicts an abrupt drop of neutron flux and neutron noise amplitude, see Fig. 4 (from $+12.5$ cm from the center). Then, in channel 7, a nearly constant neutron flux and neutron noise amplitude are estimated. On the other hand, NOISE-SN gives a different behavior such as a smoother and continuous decrease that leads to lower values. This is expected because of the possible issues of the diffusion method when applied to systems that include regions with strong variations of the material properties or regions with very low macroscopic neutron cross sections. In the CORE SIM+ simulation, the high neutron flux in channel 7 provides a high neutron leakage, which is the main reason for the very low effective multiplication factor.

In the concrete shield (see Fig. 4, between -124.5 cm and -66.5 cm, and Fig. 5, between -124.5 cm and -66.5 cm and between $+66.5$ cm and $+124.5$ cm from the center of the system), the normalized neutron flux and the neutron noise amplitude are small, but the differences between the two solvers are around one order of magnitude. Near the boundary of the system, the neutron noise calculated with NOISE-SN is affected by numerical oscillations (ray effect), which are caused by the low order of discrete ordinates (i.e., S16).

Along the channel 5-6, which is characterized by low macroscopic cross sections and where neutron detectors can be inserted during the experiment, the neutron noise amplitudes calculated with the two solvers have more significant differences in the shape of the spatial profile and in value, see Fig. 6.

The neutron noise phase estimated with the CORE SIM+ and NOISE-SN are similar, see plots at the bottom of Figs. 4, 5 and 6. Discrepancies in value are around $10^\circ - 15^\circ$, although they can be considered acceptable with respect to the overall possible phase range between 0° and 360°. A minor difference is found along the SW-NE diagonal (see Fig. 4): two small dips of the thermal noise phase, which are due to the vicinity of the channels 3-4 and 5-6, are predicted with both solvers, but one dip is much less pronounced in the CORE SIM+ simulation. In channel 5-6 (see Fig. 6), the phase of the neutron noise has a minimum which is related to the neutron noise source (more remarkable for the thermal energy group). Again, the low order of discrete ordinates used in the NOISE-SN calculation causes some numerical artifacts of the neutron noise phase in the periphery of the system (see Figs. 5 and 6).

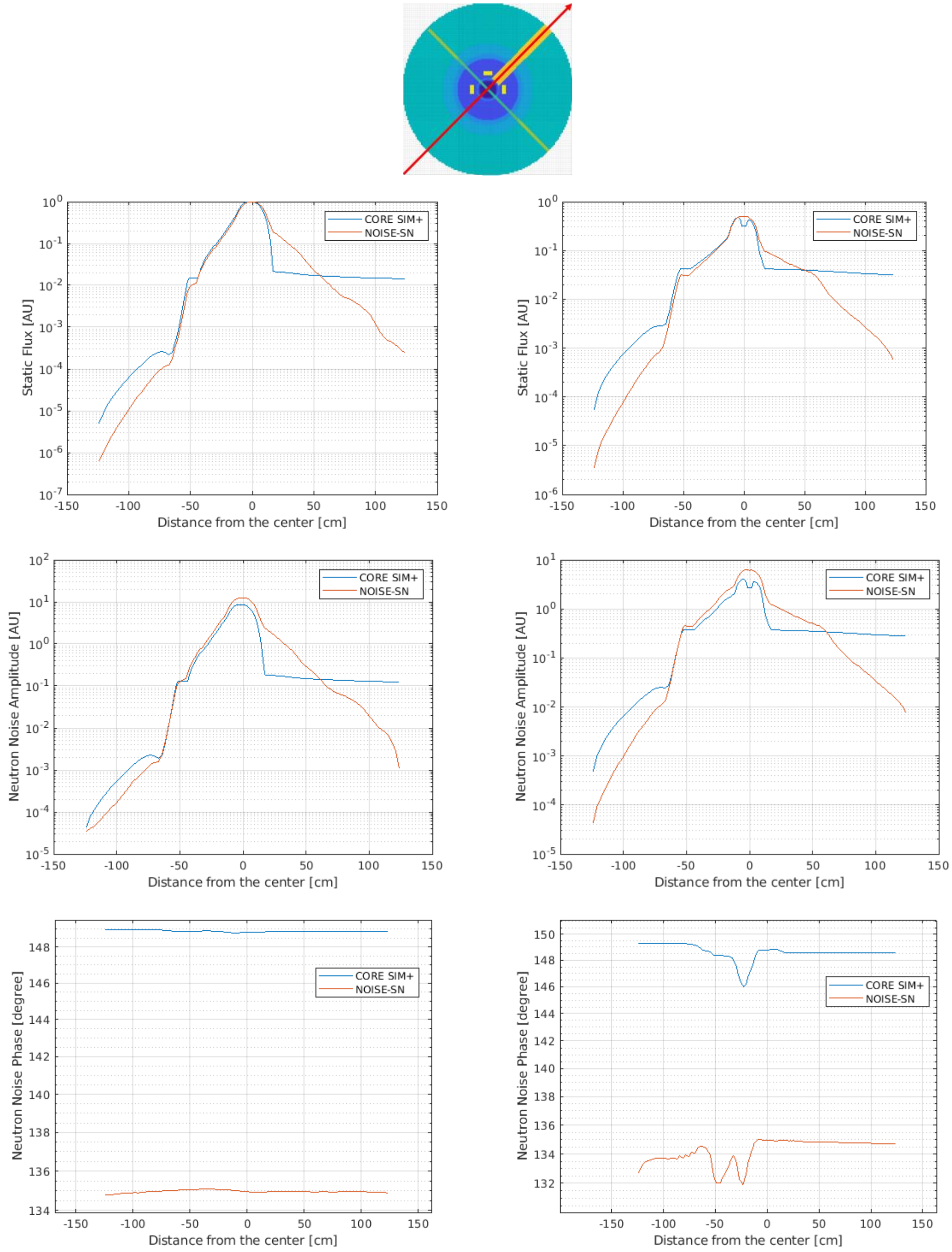

Figure 4. Comparison along the SW-NE diagonal at mid-elevation (top, red line); second row, normalized fast (left) and thermal (right) static neutron flux; third row, fast (left) and thermal (right) neutron noise amplitude (right); bottom, fast (left) and thermal (right) neutron noise phase

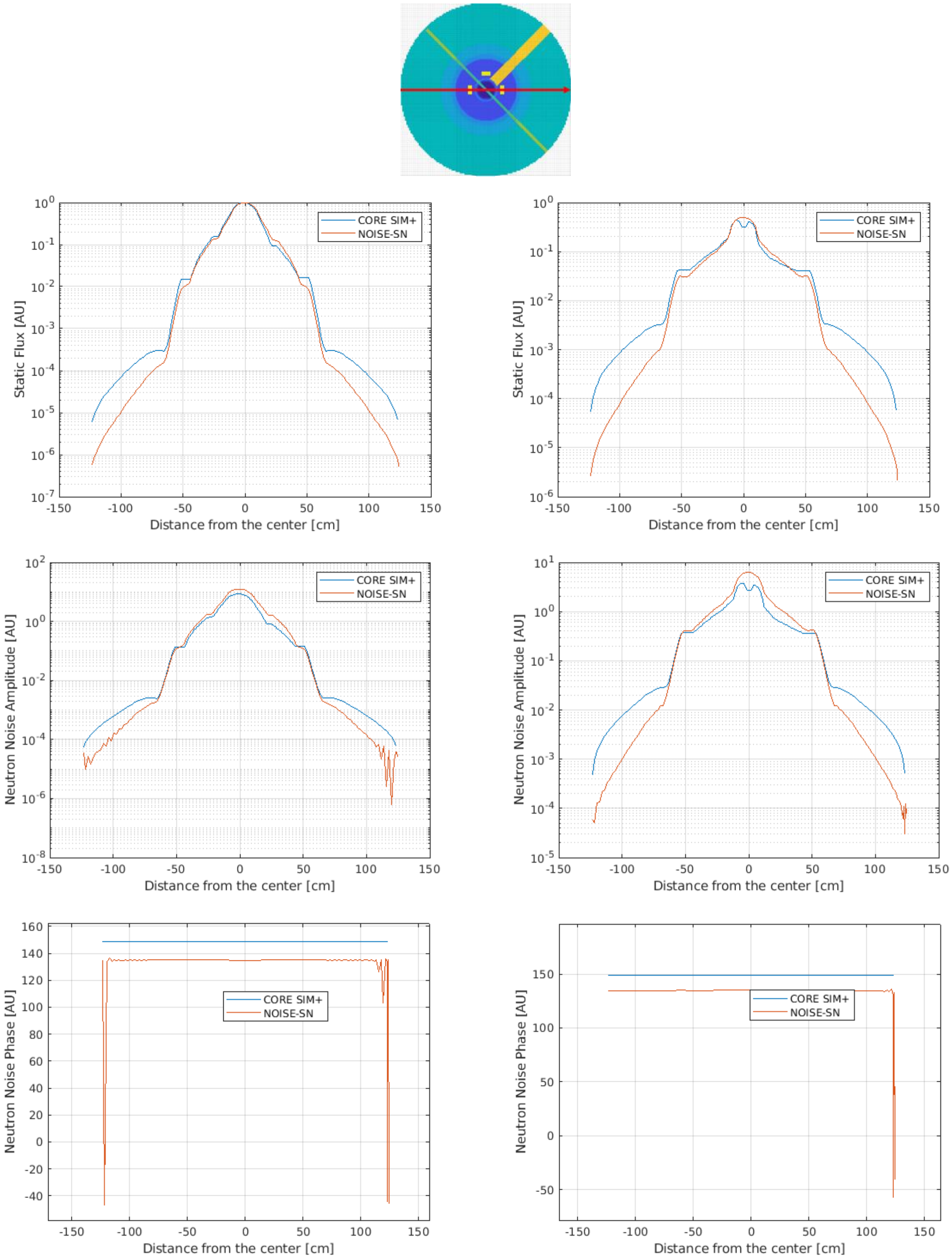

Figure 5. Comparison at mid-elevation, along the mid-horizontal (top, red line); normalized fast (left) and thermal (right) static neutron flux; third row, fast (left) and thermal (right) neutron noise amplitude (right); bottom, fast (left) and thermal (right) neutron noise phase

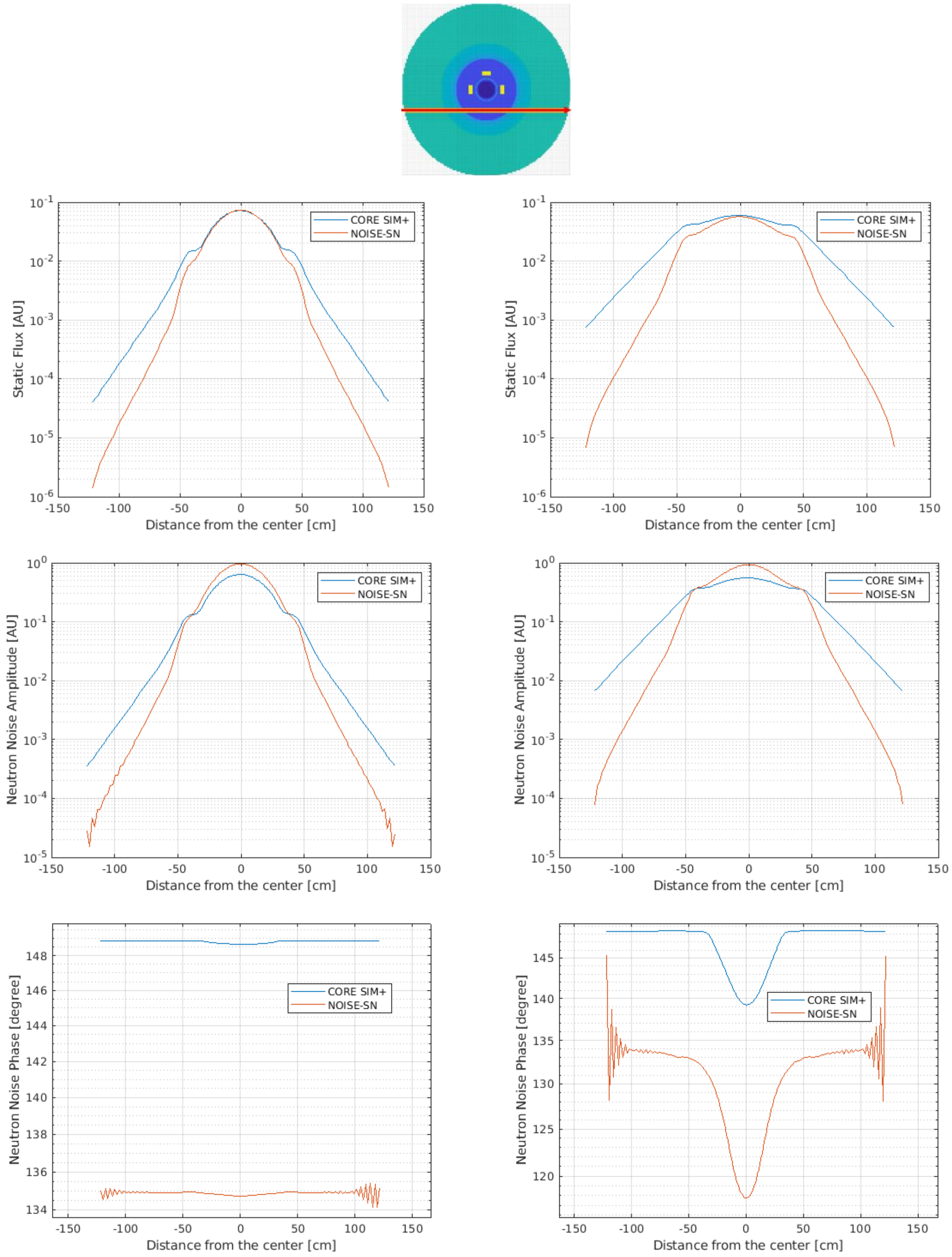

Figure 6. Comparison along channel 5-6 (top, red line); second row, normalized fast (left) and thermal (right) static neutron flux; third row, fast (left) and thermal (right) neutron noise amplitude (right); bottom, fast (left) and thermal (right) neutron noise phase

5. CONCLUSIONS

A neutron noise experiment in the research reactor AKR-2 is simulated in the frequency domain with the diffusion-based solver CORE SIM+ and the discrete ordinates solver NOISE-SN. The neutron noise is induced by a rotating thermal neutron absorber, which is equivalent to an absorber of variable strength, placed close to the reactor core. The modelling of AKR-2 reactor is demanding because of the complex and heterogeneous design. The same set of two-energy group macroscopic neutron cross sections with isotropic scattering is used for both CORE SIM+ and NOISE-SN calculations. From the preliminary analysis presented in this work, the diffusion and the discrete ordinates method evaluate similar spatial distributions of the static neutron flux and neutron noise in most of the system. However, discrepancies may be significant in regions with strong material variations and low macroscopic cross sections where the diffusion approximation is expected to be inaccurate, such as the channels available for experiments and measurements. Future work is necessary to investigate the impact of the order of discrete ordinates, anisotropic scattering, and finer energy discretization on the NOISE-SN calculations. In addition, adaptation of the macroscopic cross section and diffusion coefficients can be considered to improve the CORE SIM+ predictions. These results will be compared with the neutron noise measurements from the experiment carried out in AKR-2.

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