ON THE MODELLING OF FUEL ASSEMBLY VIBRATIONS USING COARSE MESH APPROACHES

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ABSTRACT

In this paper, the stationary fluctuations of the neutron flux as compared to its static component, also called the neutron noise, induced by the vibrations of a single fuel assembly in a commercial Pressurized Water Reactor, are considered. The purpose of the work is to investigate, using two-group diffusion theory and for one-dimensional systems, whether coarse mesh approaches are able to reproduce the global behavior of the neutron noise. It is demonstrated that the neutron noise can be faithfully reconstructed after assembly homogenization by coarse mesh simulators by introducing four noise sources only. Those noise sources are positioned at the interface between the moving fuel assembly and its non-moving neighbors. The noise sources are defined in each of the two energy groups. The strengths of the noise sources nevertheless need to be adjusted in order to properly reproduce the reference solution. Because of the four introduced noise sources, four normalization conditions are necessary. Several conditions are investigated in this work. The most practical alternative is to normalize the neutron noise sources in the homogenized configuration to the same reactivity effect as the one of the neutron noise source in the non-homogenized configuration. The two additional normalization conditions are thereafter chosen by forcing the homogenized solution to be scaled to the heterogeneous solution for each of the two energy groups. Choosing two different spatial points in the direct vicinity of the vibrating fuel assembly was demonstrated to provide a more robust reconstructed neutron noise.

KEYWORDS: neutron noise, fuel assembly vibration, fine/coarse mesh modelling, equivalence

1. INTRODUCTION

The modelling of the effect of small stationary fluctuations onto the neutron flux received in recent years a renewed interest by the community, as demonstrated in, e.g., the Horizon 2020 CORTEX project [1]. One area of focus is the vibration of fuel assemblies. Such vibrations are described by the displacement of all fuel pins belonging to the vibrating fuel assemblies. On the neutronic side, it was earlier demonstrated that the vibrations are represented by perturbations of macroscopic cross-sections introduced at the vibrating boundaries between each of the pins and the surrounding coolant (see, e.g., [2, 3]). Typically, due to the size of the reactor cores being investigated, the modelling of vibrating fuel assemblies has been considered after homogenization. The reactor cores are represented by piece-wise large homogeneous regions of ca. 10-20 cm of characteristic size. The perturbations of macroscopic cross-sections are then introduced at the boundaries of the homogeneous regions that belong to a vibrating fuel assembly (see, e.g., [4, 5, 6]). This paper aims at investigating whether the effect of vibrating fuel assemblies can be faithfully modelled in such coarse mesh approaches, as compared to the modelling at the pin level considered hereafter as the reference.

The paper is structured as followed. Section 2 describes the system considered in this analysis, as well as the modelling framework used. The modelling of vibrating fuel assemblies is described at the pin level (referred to as fine mesh modelling) and at the fuel assembly level (referred to as coarse mesh modelling). In Section 3, the results of the simulations are presented. Two types of configurations are considered: heterogeneous systems, where the heterogeneities at the pin level are explicitly represented, and homogenized systems, where classical homogenization principles are first applied. In both configurations, the effect of vibrating fuel assemblies is modelled using different approximations. Section 4 summarizes the main conclusions of the paper and gives some recommendations for the coarse mesh modelling of vibrating fuel assemblies.

2. MODELLING OF FUEL ASSEMBLY VIBRATIONS

A one-dimensional heterogeneous model representative of a commercial Pressurized Water Reactor (PWR) is used in this work. The system is made of 15 fuel assemblies, each containing 17 fuel pins. The fuel pins are 1 cm in diameter and the fuel pin pitch is 1.25 cm. The system is surrounded by a reflector having a thickness of 21.25 cm. The total size of the system is thus 361.25 cm. All calculations are performed in two-group diffusion theory. It was demonstrated in [7] that, even for pin modelling, diffusion theory is able to correctly reproduce the induced neutron noise as compared to transport theory, as long as a sufficient fine mesh is used. The reference pin calculations reported hereafter were thus carried out in diffusion theory using a fine mesh.

2.1. Fine mesh representation of vibrating fuel assemblies

When representing the reactor at the pin level, the vibrations of fuel pins can be seen as the displacement of each boundary between the pins and their surrounding media. Using the model of Rouchon and Sanchez [8], the noise source corresponding to the sinusoidal displacement $\varepsilon(t) = d\sin(\omega_0 t)$ of the position of the boundary between two homogeneous regions from its equilibrium position b_0 can be represented, in the frequency domain and at the angular frequency of the perturbation ω_0, as:

- If $x \in [b_0 - d; b_0 + d]$:

$$\delta\Sigma_\alpha(x,\omega_0) = 2\mathrm{i}\Delta\Sigma_\alpha \cos[\omega_0 \tau(x)] \tag{1}$$

- If $x \notin [b_0 - d; b_0 + d]$:

$$\delta\Sigma_\alpha(x,\omega_0) = 0 \tag{2}$$

with

$$\tau(x) = \frac{1}{\omega_0}\arcsin\left(\frac{x-b_0}{d}\right) \tag{3}$$

In Eq. (1), i is the unit imaginary number and $\Delta\Sigma_{\alpha}$ is the difference of the macroscopic cross-sections of type α between the two regions. It should be mentioned that the displacement of a boundary induces noise sources at all multiple angular frequencies of the fundamental one ω_0, as well as at the frequency of 0 rad/s, as explained in [8]. In this work, we only concentrate on the fundamental one and on the neutron noise induced at the fundamental frequency.

In addition to the above noise source representation, the so-called ε/d model was also used [9, 2]. In this model, the noise source due to the displacement of the boundary between two homogeneous regions is given, in the frequency domain at the fundamental frequency ω_0, as:

$$\delta\Sigma_{\alpha}\left(x,\omega_0\right) = \mathcal{F}\left\{\varepsilon\left(t\right)\right\}_{\omega_0} \delta\left(x-b_0\right)\Delta\Sigma_{\alpha} \tag{4}$$

where $\mathcal{F}\left\{\varepsilon\left(t\right)\right\}_{\omega_0}$ represents the Fourier-transform of the displacement $\varepsilon\left(t\right)$ of the boundary between the two regions and δ is the Dirac delta function. It was demonstrated in [3, 4] that the noise source in the ε/d model corresponds to the first-order approximation of the noise source model defined in Eqs. (1)-(3) at the fundamental frequency ω_0. Furthermore, the induced neutron noise was demonstrated to be essentially identical between the two models.

The effect of the noise source, using the model of Eqs. (1)-(3) or the model of Eq. (4), was then estimated by the CORE SIM tool in linear theory and the frequency domain using the two-group diffusion approximation [10]. In the case of the application of the ε/d model, only the case of noise sources introduced at the outer periphery of the vibrating fuel assembly was considered, as will be further explained in Section 2.2. We will come back to the reasoning behind this modelling choice in Section 3 when all results are presented.

All pins were modelled explicitly without any spatial homogenization using a mesh size of 0.00125 cm. Both the static and dynamic solutions were estimated from CORE SIM. The system was thus represented by 511 homogeneous regions, with a mesh resulting in 289 000 nodes.

2.2. Coarse mesh representation of vibrating fuel assemblies

In the coarse mesh representation, the core was first spatially homogenized. The fine mesh CORE SIM static solution was used to spatially average the macroscopic cross-sections and diffusion coefficients on each of the fuel assemblies.

The noise source was then modelled using the ε/d model earlier described. There is nevertheless one fundamental difference between the application of the ε/d model at the fine mesh level and at the core mesh level. In the fine mesh representation, a noise source is introduced at each of the boundaries between the vibrating fuel pins and the surrounding coolant. In the coarse mesh representation, the noise source is only defined at the outer boundaries of the homogenized region representing the vibrating fuel assembly. In the fine mesh case, there are thus 17x2 noise sources introduced with respect to space, whereas in the coarse mesh case, only 2 noise sources are introduced.

The same mesh of 0.00125 cm was used for the CORE SIM coarse mesh simulations as for the fine mesh simulations, both for the static and dynamic solutions. The differences between the fine mesh and coarse mesh simulations thus entirely lie with the use of homogenized cross-sections at the fuel assembly level in the coarse mesh representation, as compared with the explicit modelling of the fuel pins in the fine mesh representation. This means that the system was represented by 17 homogeneous regions, even if the mesh resulted in 289 000 nodes.

3. RESULTS

In a two-group representation, it could be demonstrated that the neutron noise in linear theory and in the frequency domain is generically given as [11]:

$$\begin{bmatrix}\delta\phi_1\left(x,\omega\right)\\ \delta\phi_2\left(x,\omega\right)\end{bmatrix}=\begin{bmatrix}\int_V\left[G_{1\to1}\left(x,x',\omega\right)S_1\left(x',\omega\right)+G_{2\to1}\left(x,x',\omega\right)S_2\left(x',\omega\right)\right]dx'\\ \int_V\left[G_{1\to2}\left(x,x',\omega\right)S_1\left(x',\omega\right)+G_{2\to2}\left(x,x',\omega\right)S_2\left(x',\omega\right)\right]dx'\end{bmatrix} \tag{5}$$

with

$$\begin{bmatrix}S_1\left(x',\omega\right)\\ S_2\left(x',\omega\right)\end{bmatrix}=\phi_r\left(x'\right)\delta\Sigma_r\left(x',\omega\right)+\phi_a\left(x'\right)\begin{bmatrix}\delta\Sigma_{a,1}\left(x',\omega\right)\\ \delta\Sigma_{a,2}\left(x',\omega\right)\end{bmatrix}+\phi_f\left(x',\omega\right)\begin{bmatrix}\delta\nu\Sigma_{f,1}\left(x',\omega\right)\\ \delta\nu\Sigma_{f,2}\left(x',\omega\right)\end{bmatrix} \tag{6}$$

In Eq. (5), $G_{i\to j}\left(x,x',\omega\right)$ is the Green's function giving the neutron noise in the energy group j at the position x induced by a point-like source in the energy group i at the position x', i.e., when $S_i\left(x,\omega\right)=\delta\left(x-x'\right)$. In Eq. (6), the elements of the vector $\phi_r\left(x'\right)$ and of the matrix $\phi_a\left(x'\right)$ are made of the static flux values. For the matrix $\phi_f\left(x',\omega\right)$, in addition to the static flux values, the kinetic parameters and the frequency of the perturbation are also used. The perturbations in the cross-sections $\delta\Sigma_r\left(x',\omega\right)$, $\delta\Sigma_{a,1}\left(x',\omega\right)$, $\delta\Sigma_{a,2}\left(x',\omega\right)$, $\delta\nu\Sigma_{f,1}\left(x',\omega\right)$ and $\delta\nu\Sigma_{f,2}\left(x',\omega\right)$ are defined using either the model of Eqs. (1)-(3) or the model of Eq. (4).

When using the ε/d model given by Eq. (4), one notices from Eq. (5) that the perturbations of the macroscopic cross-sections lead to the problem entirely defined by four noise sources only: S_1 on each side of the vibrating structure and S_2 on each side of the vibrating structure. In the case of the homogenized problem in particular, those noise sources are introduced at the boundary of the region representing the vibrating fuel assembly. In a one-dimensional representation, this means that only four noise sources are introduced. One of the questions this paper attempts to address is whether the neutron noise induced by 17 vibrating fuel pins can be faithfully represented by four noise sources only in a homogenized coarse mesh representation.

3.1. Heterogeneous systems

In order to address the answer to the above question, the reference solution in the heterogeneous representation of the system was first considered using the cross-section perturbation model given by Eqs. (1)-(3). Thereafter, only four noise sources introduced at the outer boundary of the extreme moving pins in the vibrating fuel assembly were considered using the ε/d cross-section perturbation model given by Eq. (4). Although the ultimate goal is to investigate whether four noise sources can faithfully reproduce the induced neutron noise after spatial homogenization (which will be the purpose of Section 3.2), it was deemed necessary to consider the case before spatial homogenization as well.

As Eq. (4) demonstrates, each of the four noise sources is a complex number. The possibility to reproduce the exact neutron noise is thus equivalent to define such four complex numbers. Four conditions are thus necessary to fully define the search procedure. These conditions were defined by forcing the solution to be equal to the actual solution at the two locations of the introduced noise sources, for both the fast and the thermal group. The corresponding results are presented in Fig. 1. As the plots demonstrate, keeping the heterogeneous description of the system, four noise sources are sufficient to fully describe the effect of the induced neutron noise. In the thermal group in particular, for which the mean free path gives rise to local effects at the pin level, the superposition of those effects, i.e., the global behavior of the neutron noise, is properly described by the four noise sources only.

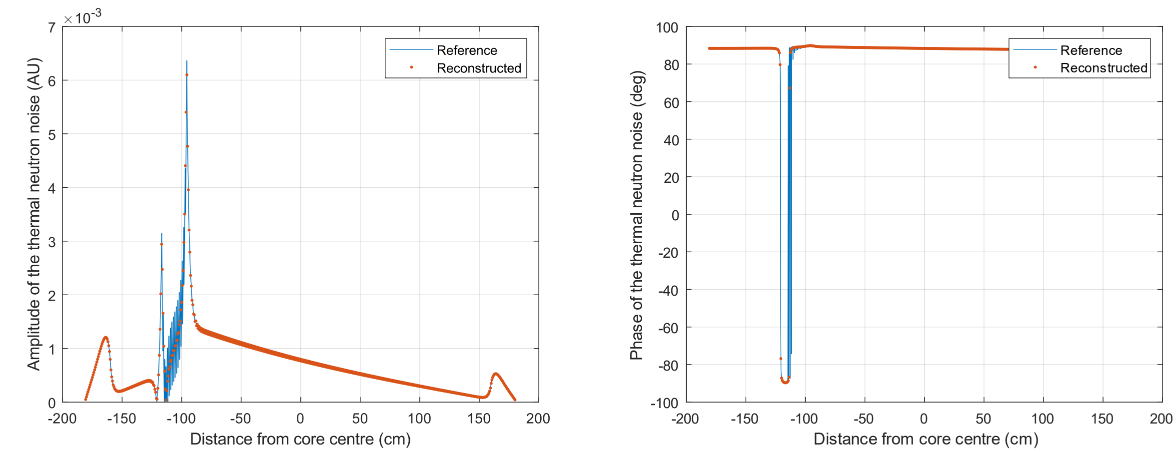

Figure 1. Fast (upper figures) and thermal (lower figures) neutron noise induced by 17 vibrating fuel pins (labelled "reference") and 4 noise sources (labelled "reconstructed"). The amplitude of the neutron noise is given on the left figures, whereas the phase of the neutron noise is given on the right figures. Both solutions were estimated in the heterogeneous problem.

The above results also indicate that four normalization conditions to the reference solution are necessary. Instead of using the reference noise solutions directly in all four conditions, two of the conditions are now replaced by forcing the indued neutron noise to have the same reactivity effect in the reconstructed and reference solutions. Using first-order perturbation theory, this reads in the frequency domain:

- For the fast group, as:

$$\delta\rho_{1,\delta\phi}\left(\omega\right)=\frac{1}{A_0G_0\left(\omega\right)}\int_V\frac{1}{v_1}\phi_{1,0}^{+}\left(x\right)\delta\phi_1^{\text{reconstructed}}\left(x,\omega\right)dx=\frac{1}{A_0G_0\left(\omega\right)}\int_V\frac{1}{v_1}\phi_{1,0}^{+}\left(x\right)\delta\phi_1^{\text{reference}}\left(x,\omega\right)dx \quad (7)$$

- For the thermal group, as:

$$\delta\rho_{2,\delta\phi}\left(\omega\right)=\frac{1}{A_0G_0\left(\omega\right)}\int_V\frac{1}{v_2}\phi_{2,0}^{+}\left(x\right)\delta\phi_2^{\text{reconstructed}}\left(x,\omega\right)dx=\frac{1}{A_0G_0\left(\omega\right)}\int_V\frac{1}{v_2}\phi_{2,0}^{+}\left(x\right)\delta\phi_2^{\text{reference}}\left(x,\omega\right)dx \quad (8)$$

with

$$A_0=\int_V\left[\frac{1}{v_1}\phi_{1,0}^{+}\left(x\right)\phi_{1,0}\left(x\right)+\frac{1}{v_2}\phi_{2,0}^{+}\left(x\right)\phi_{2,0}\left(x\right)\right]dx \quad (9)$$

In the above equations, $\phi_{i,0}$ and $\phi_{i,0}^{+}$ denotes the static neutron flux and its adjoint, respectively, in the energy group i and v_i is the corresponding group-averaged neutron speed. $G_0\left(\omega\right)$ is the zero-power reactor transfer function, defined as:

$$G_0\left(\omega\right)=\frac{1}{\mathrm{i}\omega\left(\Lambda_0+\frac{\beta}{\mathrm{i}\omega+\lambda}\right)} \quad (10)$$

where i is the unit imaginary number. Eqs. (7) and (8) were obtained by using the property of orthogonality between the adjoint functions and the shape functions associated to the neutron noise [11]. Using the two normalization conditions expressed by Eqs. (7) and (8) and forcing the solution to be equal to the actual solution at the two locations of the introduced noise sources (noise source closest to the core periphery for the fast group and noise source closest to the core center for the thermal group) was demonstrated to give the correct reconstructed solution.

Since the normalization to the same reactivity effect leads to a proper reconstruction, the estimation of the reactivity effect could be performed using the introduced perturbation directly instead of its effect on the induced neutron noise. Using first-order perturbation theory, this reads in the frequency domain:

- For the fast group, as:

$$\begin{aligned}&\delta\rho_{1,\delta\Sigma}\left(\omega\right)\\&=\frac{1}{F_0}\int_V\left\{\left[\frac{\delta\upsilon\Sigma_{f,1}^{\text{reconstructed}}\left(x,\omega\right)}{k_{eff}}-\delta\Sigma_{a,1}\left(x,\omega\right)-\delta\Sigma_{rem}\left(x,\omega\right)\right]\phi_{1,0}^{+}\left(x\right)\phi_{1,0}\left(x\right)+\frac{\delta\upsilon\Sigma_{f,2}^{\text{reconstructed}}\left(x,\omega\right)}{k_{eff}}\phi_{1,0}^{+}\left(x\right)\phi_{2,0}\left(x\right)\right\}dx\\&=\frac{1}{F_0}\int_V\left\{\left[\frac{\delta\upsilon\Sigma_{f,1}^{\text{reference}}\left(x,\omega\right)}{k_{eff}}-\delta\Sigma_{a,1}\left(x,\omega\right)-\delta\Sigma_{rem}\left(x,\omega\right)\right]\phi_{1,0}^{+}\left(x\right)\phi_{1,0}\left(x\right)+\frac{\delta\upsilon\Sigma_{f,2}^{\text{reference}}\left(x,\omega\right)}{k_{eff}}\phi_{1,0}^{+}\left(x\right)\phi_{2,0}\left(x\right)\right\}dx\end{aligned}$$

(11)

- For the thermal group, as:

$$\begin{aligned}&\delta\rho_{2,\delta\Sigma}\left(\omega\right)\\&=\frac{1}{F_0}\int_V\left\{\delta\Sigma_r^{\text{reconstructed}}\left(x,\omega\right)\phi_{2,0}^{+}\left(x\right)\phi_{1,0}\left(x\right)-\delta\Sigma_{a,2}^{\text{reconstructed}}\left(x,\omega\right)\phi_{2,0}^{+}\left(x\right)\phi_{2,0}\left(x\right)\right\}dx \quad (12)\\&=\frac{1}{F_0}\int_V\left\{\delta\Sigma_r^{\text{reference}}\left(x,\omega\right)\phi_{2,0}^{+}\left(x\right)\phi_{1,0}\left(x\right)-\delta\Sigma_{a,2}^{\text{reference}}\left(x,\omega\right)\phi_{2,0}^{+}\left(x\right)\phi_{2,0}\left(x\right)\right\}dx\end{aligned}$$

with

$$F_0 = \frac{1}{k_{eff}} \int_V \left[\upsilon \Sigma_{f,1,0}\left(x\right) \phi_{1,0}^{+}\left(x\right) \phi_{1,0}\left(x\right) + \upsilon \Sigma_{f,2,0}\left(x\right) \phi_{1,0}^{+}\left(x\right) \phi_{2,0}\left(x\right) \right] dx \tag{13}$$

Using the two normalization conditions expressed by Eqs. (11) and (12) and forcing the solution to be equal to the actual solution at the two locations of the introduced noise sources (noise source closest to the core periphery for the fast group and noise source closest to the core center for the thermal group) was demonstrated to give the correct reconstructed solution as well. It should be mentioned that the group-wise reactivity effects estimated by Eqs. (7) and (8), and by Eqs. (11) and (12), respectively, are different. Only their sums are identical, i.e., $\delta\rho_{1,\delta\phi}\left(\omega\right) + \delta\rho_{2,\delta\phi}\left(\omega\right) = \delta\rho_{1,\delta\Sigma}\left(\omega\right) + \delta\rho_{2,\delta\Sigma}\left(\omega\right)$ whereas $\delta\rho_{1,\delta\phi}\left(\omega\right) \neq \delta\rho_{1,\delta\Sigma}\left(\omega\right)$ and $\delta\rho_{2,\delta\phi}\left(\omega\right) \neq \delta\rho_{2,\delta\Sigma}\left(\omega\right)$. Compared to the normalization conditions given by Eqs. (7) and (8), Eqs. (12) and (13) have the clear advantage that the reference solution throughout the core is not needed. Only the noise sources in the exact problem formulation and in the equivalent four noise source formulation are required. It should nevertheless be mentioned that Eqs. (11) and (12), as well as Eqs. (7) and (8), only provide two of the four necessary normalization conditions. The two additional normalization conditions should be provided by fitting the reconstructed solution to the reference one at two spatial points.

For the sake of completeness, additional simulations (not reported here) showed that using less than four normalization conditions does not allow reconstructing the true solution.

3.2. Homogenized systems

Based on the results reported in Section 3.1 for the actual heterogeneous system, one wants to verify that four noise sources are also sufficient to reproduce the overall behavior of the neutron noise after assembly homogenization. Since using a normalization to the same reactivity effect of the neutron noise source does not require any knowledge of the reference solution for such a normalization and is thus clearly advantageous, two of the necessary conditions are provided using Eqs. (11)-(13). The two additional conditions are then given by the normalization to the reference solution. In Fig. 2, the reconstructed neutron noise in the homogenized problem is compared to the reference neutron noise in the heterogeneous problem where the two additional conditions were defined by forcing the solution to be equal to the actual solution at the two locations of the introduced noise sources (noise source closest to the core periphery for the fast group and noise source closest to the core center for the thermal group). As demonstrated in the figure, the agreement between both solutions at the large scales is excellent. As noticed in Section 3.1 for the heterogeneous configuration, the local changes of the thermal neutron noise in the vibrating fuel assembly cannot be reproduced with four noise sources only, but the spatially averaged neutron noise is correctly reproduced even in the vibrating fuel assembly.

Additional simulations nevertheless revealed that the ability to correctly reproduce the global behavior of the reference solution strongly depends on the choice of the two normalization conditions in addition to the ones given by Eqs. (11) and (12). More precisely, it was demonstrated that the spatial points chosen for the normalization need to be in the neighborhood of the vibrating fuel assembly. Furthermore, choosing two different spatial points, one on each side of the vibrating fuel assembly, seem to provide more robust results. Using either the reference fast and thermal neutron noise, respectively, for each of the conditions, or the thermal neutron noise for both conditions, leads to a correct reconstructed neutron noise in both energy groups.

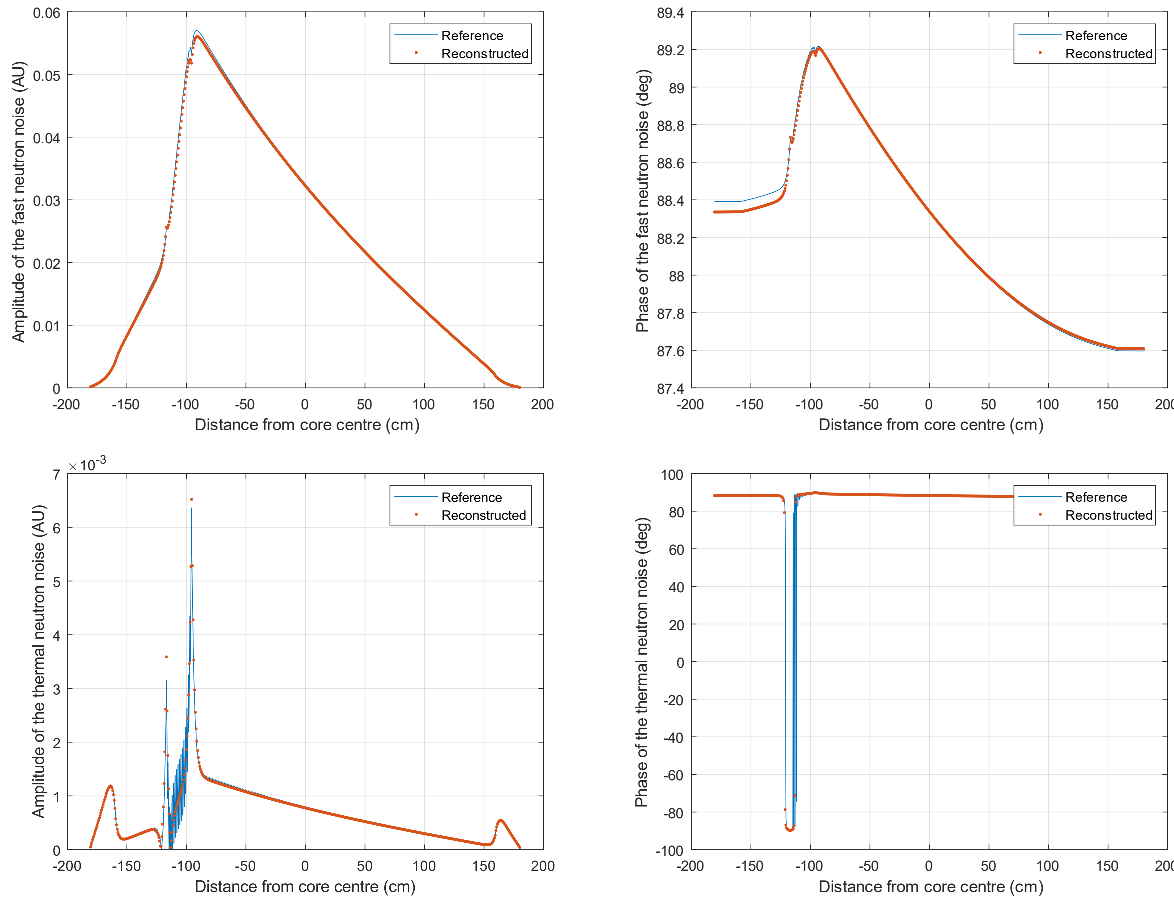

Figure 2. Fast (upper figures) and thermal (lower figures) neutron noise induced by 17 vibrating fuel pins (labelled "reference") and 4 noise sources (labelled "reconstructed"). The amplitude of the neutron noise is given on the left figures, whereas the phase of the neutron noise is given on the right figures. The reference solution was estimated in the actual heterogeneous problem, whereas the reconstructed solution was obtained after assembly homogenization.

4. CONCLUSIONS

In this paper, it was demonstrated that the neutron noise induced by the vibrations of a fuel assembly made of 17 pins in a commercial PWR can be faithfully reproduced at the global scales by introducing only four noise sources, both in the actual heterogeneous representation of the system and after assembly homogenization. In the homogenized system in particular and using the ε/d cross-section perturbation model, the noise sources are positioned at the interface between the vibrating fuel assembly and its non-vibrating neighbors. It is truly remarkable that such a complex phenomenon can be reproduced with only so few noise sources introduced. This also has some implications on the modelling of the neutron noise induced by vibrating fuel assemblies when using traditional coarse mesh core simulators, as this study demonstrates that, by properly choosing those four noise sources, the actual neutron noise can be correctly reproduced. It should nevertheless be emphasized that the strength of those noise sources in the ε/d model needs to be considerably adjusted as compared to the direct estimation of $\Delta\Sigma_\alpha$ carried out by simply taking the difference in the macroscopic cross-sections between two adjacent homogenized regions. This issue will be the topic of a separate investigation.

Two of the necessary normalization conditions can be obtained by ensuring that the four noise sources have the same reactivity effect as the one of the noise sources introduced by each of the 17 moving fuel pins. The two additional conditions are provided by normalizing the reconstructed solution to the reference solution, preferably in the vicinity of the vibrating fuel assembly. As the normalization to the same reactivity effect guarantees the same point-kinetic response between the reconstructed and reference solutions, the deviations from the point-kinetic response are expected to be more significant close to the vibrating fuel assembly, explaining why the two spatial points for the additional conditions should be chosen close to the vibrating fuel assembly. Although not considered in this work, it would be interesting to investigate the possibility to alternatively use, as normalization conditions, the global and local relaxation lengths of the induced neutron noise between the reconstructed and reference neutron noise.

The findings reported in this paper would also need to be complemented by transport calculations of the reference solution in the non-homogenized configuration, in order to verify that that coarse mesh results in diffusion theory are also able to reproduce the overall behavior of the transport solution.

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REFERENCES

1. C. Demazière, P. Vinai, M. Hursin, S. Kollias and J. Herb, "Overview of the CORTEX project," *Proceedings of the International Conference on the Physics of Reactors – Reactor Physics paving the way towards more efficient systems (PHYSOR2018)*, Cancun, Mexico, April 22-26, 2018 (2018).
2. A. Jonsson, H.N. Tran, V. Dykin and I. Pázsit, "Analytical investigation of the properties of neutron noise induced by vibrating absorber and control rods," *Kerntechnik*, **77** (5), pp. 371-380 (2012).
3. A. Zoia, A. Rouchon, B. Gasse, C. Demazière and P. Vinai, "Analysis of the neutron noise induced by fuel assembly vibrations," *Annals of Nuclear Energy*, **154**, 108061 (2021).
4. A. Vidal-Ferràndiz, A. Carreño, D. Ginestar, C. Demazière and G. Verdú, "Neutronic simulation of fuel assembly vibrations in a nuclear reactor," *Nuclear Science and Engineering*, **194**, pp. 1067-1078 (2020).
5. A. Mylonakis, P. Vinai and C. Demazière, "CORE SIM+: A flexible diffusion-based solver for neutron noise simulations," *Annals of Nuclear Energy*, **155**, 108149 (2021).
6. V. Verma, D. Chionis, A. Dokhane and H. Ferroukhi, "Studies of reactor noise response to vibrations of reactor internals and thermal-hydraulic fluctuations in PWRs," *Annals of Nuclear Energy*, **157**, 108212 (2021).
7. P. Vinai, H. Yi, A. Mylonakis, C. Demazière, B. Gasse, A. Rouchon, A. Zoia, A. Vidal-Ferràndiz, D. Ginestar, G. Verdú and T. Yamamoto, "Comparison of neutron noise solvers based on numerical benchmarks in a 2-D simplified UOX fuel assembly," *Proceedings of the International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering (M&C2021)*, Raleigh, North Carolina, USA, October 3-7, 2021 (2021).
8. A. Rouchon and R. Sanchez, "Analysis of vibration-induced neutron noise using one-dimension noise diffusion theory," *Proceedings of International Congress on Advances in Nuclear Power Plants (ICAPP2015)*, Nice, France, May 3-5, 2015 (2015).
9. I. Pázsit, "Control rod models and vibration induced noise," *Annals of Nuclear Energy*, **15** (7), pp. 333-346 (1988).
10. C. Demazière, "CORE SIM: A multi-purpose neutronic tool for research and education," *Annals of Nuclear Energy*, **38** (12), pp. 2698-2718 (2011).

11. I. Pázsit and C. Demazière, *Noise techniques in nuclear systems*. Chapter in the *Handbook of Nuclear Engineering*, by D. Cacuci (Ed.), ISBN 978-0-387-98150-5, Springer, Vol. 3 (2010).