COMPARISON OF NEUTRON NOISE SOLVERS BASED ON NUMERICAL BENCHMARKS IN A 2-D SIMPLIFIED UOX FUEL ASSEMBLY

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ABSTRACT

In the CORTEX project, several solvers are developed and applied to analyze neutron noise problems. They are based on Monte Carlo and deterministic (higher-order transport and diffusion) methods. For the study of their validity and limitations, an extensive verification and validation work has been undertaken and includes the simulation of numerical exercises and experiments. In the current paper the solvers are compared over two neutron noise benchmarks defined in a 2-D simplified UOX fuel assembly, with Monte Carlo used as a reference. In the two exercises, a global neutron noise source and a combination of stationary perturbations of the various cross sections are respectively prescribed. The higher-order neutron transport methods provide consistent results with respect to Monte Carlo. The calculations obtained from the diffusion-based solvers show discrepancies that can be significant, in particular close to the neutron noise source.

KEYWORDS: neutron noise, neutron transport, Monte Carlo, higher-order transport methods, neutron diffusion theory

1. INTRODUCTION

The CORTEX project aims at investigating neutron noise-based techniques for reactor core monitoring and diagnostics [1]. The strategy is to characterize and localize possible anomalies in the reactor core from the analysis of the small stationary fluctuations (i.e. the so-called neutron noise) that are observed in the measurements of the neutron flux. One of the crucial efforts required in the approach is the modelling of the reactor transfer function that allows computing the effect of perturbations of the macroscopic cross sections on the neutron flux. For this purpose, solvers based on the Monte Carlo method, the discrete ordinates method, the method of characteristics and diffusion theory have been developed and tested in the project. The study of their respective validity and limitations is an important aspect for their application. Thus, an extensive verification and validation work has been undertaken and includes simulations of both numerical benchmarks (e.g., [2-4]) and neutron noise experiments (e.g., [5]). In the current paper, the solvers are compared over two numerical exercises in which the neutron noise induced by prescribed stationary perturbations in a 2-D simplified UOX fuel assembly is calculated. After this introduction, the paper is arranged as follows. In Section 2 the solvers are presented. In Section 3 the neutron noise exercises are discussed. In Section 4 illustrative results are shown. In Section 5 conclusions are drawn.

2. NEUTRON NOISE SOLVERS

The neutron noise solvers compared in this work are:

- A stochastic solver in the Monte Carlo code TRIPOLI-4®, developed by CEA [6, 7]
- A Monte Carlo solver, developed by KU – Kyoto University [8]
- A deterministic solver based on a discrete ordinates method, developed by Chalmers University of Technology [9, 10]
- The deterministic Integro-Differential Transport - IDT lattice solver embedded in APOLLO3®, developed by CEA [2]
- The diffusion-based solver CORE SIM+, developed by Chalmers University of Technology [11]
- The diffusion-based solver FEMFFUSION, developed by UPV – Universitat Politècnica de València [3, 12]

FEMFFUSION computes the time-dependent solution of the neutron diffusion equation, while the other solvers make use of the formulation of the neutron noise equation in the frequency domain and thus calculate the neutron noise as a complex quantity.

2.1. The stochastic solver in TRIPOLI-4®

A stochastic noise solver in the frequency domain has been implemented in the development version of the Monte Carlo TRIPOLI-4® at CEA [6]. The noise equations are solved by transporting particles carrying two statistical weights, one for the real part and one for the imaginary part of the noise field. Particle flights are sampled from an exponential distribution, as for the regular Boltzmann equation, whereas the collision events are modified by the presence of complex operators in the noise equations (an additional imaginary absorption cross section and a complex delayed neutron yield). Such terms are dealt with by correspondingly modifying the particle weights. We refer to [6] and [7] for a thorough description of the implemented algorithms. If required, the noise source term is preliminarily computed by running a power iteration and sampling from the frequency-dependent distributions.

2.2. The KU Monte Carlo solver

The algorithm that is adopted in the Monte Carlo solver developed by Kyoto University is fundamentally the same as that in TRIPOLI-4®. The main difference from TRIPOLI-4® is that a special term in the frequency-domain neutron noise transport equation, $i\omega\phi/v$, is dealt with by changing a complex-valued particle weight continuously during each flight distance. However, this treatment for the special term is mathematically equivalent to the method in TRIPOLI-4®. The details of the Monte Carlo algorithm are presented in [8].

2.5. The Chalmers transport solver

The neutron noise transport solver developed by Chalmers is based on the finite diamond difference method for the spatial discretization, the discrete ordinates method for the angular discretization, and the multi-energy group formalism. The Chebyshev-Legendre quadrature is used to construct the scalar neutron flux form the angular neutron flux. The iterative scheme is accelerated using a Coarse Mesh Finite Difference – CMFD technique. Considering a critical nuclear system with a perturbation described as small fluctuations of the macroscopic neutron cross sections, the solver first calculates the neutron flux and the multiplication factor associated with the static problem. Then the neutron noise equations are solved in the frequency domain, so that the amplitude and the phase of the neutron noise are determined according to the prescribed neutron noise source and the estimated static solution.

2.3. The IDT lattice solver in APOLLO3®

A deterministic noise equation solver in the frequency domain has been implemented in IDT, the lattice solver in APOLLO3® based on the Sn discrete ordinates method and on the method of short characteristics (MOSC). The standard iteration loops are applied to the fission source (but the production operator is now complex) and to the scattering source as customary, and an iteration loop between the real and imaginary parts of the neutron noise equation is added: details can be found in [2]. Thus, the standard one-group transport solver methods can be used, and one can consequently benefit from all numerical methods already implemented in APOLLO3®. At present, the noise solver of IDT is capable of dealing with homogeneous Cartesian geometries. If needed, the noise source is computed by running a power iteration.

2.4. CORE SIM+

The neutron noise simulator CORE SIM+ relies on a two-energy group diffusion model with one family of precursors of delayed neutrons. The numerical scheme can make use of uniform or non-uniform meshes for the spatial discretization of the neutron balance equations. The neutron noise source is modelled as small fluctuations of macroscopic neutron cross sections in a critical nuclear system, and the calculation of the induced neutron noise consists of two steps. In the first step, the static neutron equations are solved via the power iteration method accelerated by Chebyshev polynomials or by a Jacobian Free Newton-Krylov technique. In the second step, the neutron noise equations are solved in the frequency domain, using the static neutron flux and the multiplication factor previously evaluated and assuming no deviation of the perturbed system from criticality. The numerical solution of the linear systems generated from the power iteration algorithm and from the neutron noise equations is given by the GMRES method combined with an ILU(0) or SGS preconditioner.

2.6. FEMFFUSION

FEMFFUSION is an open-source general time-domain code that solves the multigroup time-dependent neutron diffusion equation developed by UPV [3, 12]. This code uses a spatial discretization based on the continuous Galerkin finite element method (FEM) and it is able to deal with any type of geometry and any problem dimension (1D, 2D and 3D problems). FEMFFUSION can solve any type of perturbation of the reactor steady state as rod ejections. Also, it is possible to solve generic changes in the reactor inserted as a custom set of time-domain cross-sections. Recently, it was updated to be able to solve neutron noise perturbations in the time domain as generic absorbers of variable strength and vibrating fuel assemblies [3].

Neutron noise problems require that the numerical solvers used are set to low tolerances in order to accurately detect small fluctuations in the neutron flux. The code is openly available at [12].

3. BENCHMARKS FOR THE COMPARISON OF THE SOLVERS

Two neutron noise benchmarks defined in [7] are used for the comparison of the solvers. The system configuration is a 2-dimensional simplified UOX fuel assembly for Pressurized Water Reactors (PWRs). The simplified fuel assembly together with the reference computational spatial grid is shown in Figure 1. The system includes 264 homogeneous square fuel pins and 25 homogeneous water holes. The size of the system is $21.58\ cm \times 21.58\ cm$, the size of the fuel pin is $0.7314\ cm \times 0.7314\ cm$, and the size of the water hole is $1.26\ cm \times 1.26\ cm$. The assembly is surrounded by a water blade of thickness equal to $0.08\ cm$. The boundary conditions are reflective. When applying the discrete ordinates method, the S_{32} approximation is chosen. The nuclear data are generated with respect to 2 energy groups, and scattering is assumed to be isotropic. All the solvers, including the Monte Carlo ones, use the same set of pre-generated cross sections, see Table I.

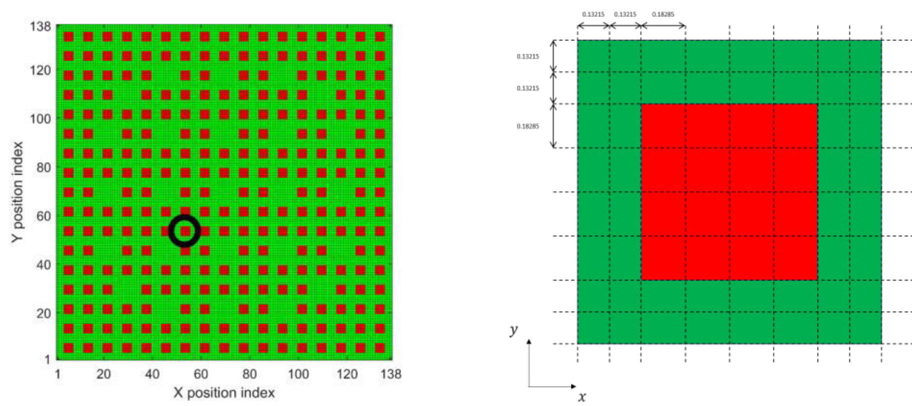

Figure 1. Simplified UOX fuel assembly (left) and computational spatial grid for the fuel cell (right); fuel pins are in red, water region is in green, and the perturbed fuel pin is highlighted with a black circle

The neutron noise exercises are defined as the perturbation of the properties of one fuel pin in the assembly. As indicated in Figure 1, counting the columns and the rows of fuel pins from the lower left corner, the perturbed fuel pin is identified by column 7 and row 7.

In *exercise 1*, the fluctuation of the properties of the fuel pin is assumed to give a resulting neutron noise source whose fast and thermal components in the frequency domain are equal to 0 and $-1+i$, respectively. The frequency of the neutron noise source is 3 Hz.

In *exercise 2*, the neutron noise source is a combination of different fluctuations associated with the macroscopic neutron cross sections of the fuel pin. The perturbed cross sections are expressed as:

$$\Sigma_{t,g} = \Sigma_{t,g,0} + \delta\Sigma_{t,g} = \Sigma_{t,g,0} + 0.041\Sigma_{t,g,0}\cos(\omega_0 t)\ , \tag{1}$$

$$\Sigma_{s,g\to g'} = \Sigma_{s,g\to g',0} + \delta\Sigma_{s,g\to g'} = \Sigma_{s,g\to g',0} + 0.034\Sigma_{s,g\to g',0}\cos(\omega_0 t)\ , \tag{2}$$

$$\Sigma_{f,g} = \Sigma_{f,g,0} + \delta\Sigma_{f,g} = \Sigma_{f,g,0} + 0.021\Sigma_{f,g,0}\cos(\omega_0 t)\ , \tag{3}$$

The quantities $\Sigma_{t,g,0}$ and $\Sigma_{f,g,0}$ are respectively the static total cross section and the static fission macroscopic cross section, for the $g-th$ energy neutron group, with $g = 1,2$. The quantity $\Sigma_{s,g\to g',0}$ represents the static group-to-group isotropic scattering matrix from the $g-th$ energy neutron group to the $g'-th$ energy neutron group, with g and $g' = 1,2$. The perturbations $\delta\Sigma_{t,g}$, $\delta\Sigma_{s,g\to g'}$ and $\delta\Sigma_{f,g}$ are cosine functions with angular frequency ω_0, that is chosen to be equal to 2π rad/s, i.e. the frequency of the neutron noise source is 1 Hz.

Table I. Nuclear data for the fuel assembly: group 1 is for the fast neutrons and group 2 is for the thermal neutrons.

Data	Homogeneous square fuel pin	Homogeneous water hole and water blade
Total cross section, group 1 (cm^{-1})	0.3779	0.25411
Total cross section, group 2 (cm^{-1})	0.55064	1.2182
Absorption cross section, group 1 (cm^{-1})	0.025755	0.00079457
Absorption cross section, group 2 (cm^{-1})	0.15788	0.029316
Fission cross section, group 1 (cm^{-1})	0.0057671	0.0
Fission cross section, group 2 (cm^{-1})	0.10622	0.0
Average number of neutrons per fission event, group 1	2.59068	0.0
Average number of neutrons per fission event, group 2	2.59068	0.0
Scattering cross section, group 1 to group 2 (cm^{-1})	0.00086471	0.028124
Velocity, group 1 (cm. s^{-1})	1.82304E+07	
Velocity, group 2 (cm. s^{-1})	4.13067E+05	
Fraction of delayed neutrons (pcm)	535	
Precursor decay time (s^{-1})	0.0851	

4. RESULTS

The quantities chosen for the comparison of the solvers are the neutron flux and the multiplication factor in the static configuration without perturbations, and the relative amplitude and phase of the neutron noise calculated in the two exercises. In the post-processing phase after the calculations, the fuel assembly is considered without the surrounding water blade. The static neutron flux for the energy group g is normalized by the fast neutron flux computed in the first computational cell located at the left-bottom corner of the fuel assembly without the water blade (see Figure 1). A similar procedure is used for the neutron noise amplitude. In addition, the normalized neutron noise amplitude is divided by the normalized static neutron flux for the same energy group g, so that relative values of the neutron noise are determined. For illustration, the results are taken along the main diagonal of the fuel assembly that crosses the perturbed fuel pin. The TRIPOLI-4® simulations are selected as the reference.

The calculated values of the multiplication factor of the static configuration are summarized in Table II. The KU Monte Carlo solver, the IDT solver embedded in APOLLO3®, and the discrete ordinates solver developed by Chalmers predict values close to the multiplication factor obtained from TRIPOLI-4®. The diffusion-based codes CORE SIM+ and FEMFFUSION exceed the reference by more than 1000 pcm. The fast and thermal static neutron flux together with the relative differences between the solvers and TRIPOLI-

4® are shown in Figure 2 and 3, respectively. The relative differences associated with the KU Monte Carlo solver and with the deterministic higher-order transport methods are between $\pm\sim1\%$. The relative differences associated with the diffusion solvers are between $+1\%$ and -4% for the fast neutron flux and between $+2\%$ and -6% for the thermal neutron flux. The larger discrepancies between the diffusion and Monte Carlo calculations are found in the water holes and in the middle of the moderator regions between the fuel pins. Although not included in the plots, the standard deviations associated with the Monte Carlo results are sufficiently small to ensure the accurate estimation of the neutron fluxes.

Table II. Multiplication factor: comparison between codes over the static configuration.

Solvers	k_{eff}	Difference [pcm]
TRIPOLI-4®	0.99912 ± 8 pcm	Reference
KU Monte Carlo solver	0.99919 ± 7 pcm	7
APOLLO-3®	0.99784	-128
Chalmers SN solver	0.99996	84
CORE SIM+	1.01309	1397
FEMFFUSION	1.01367	1485

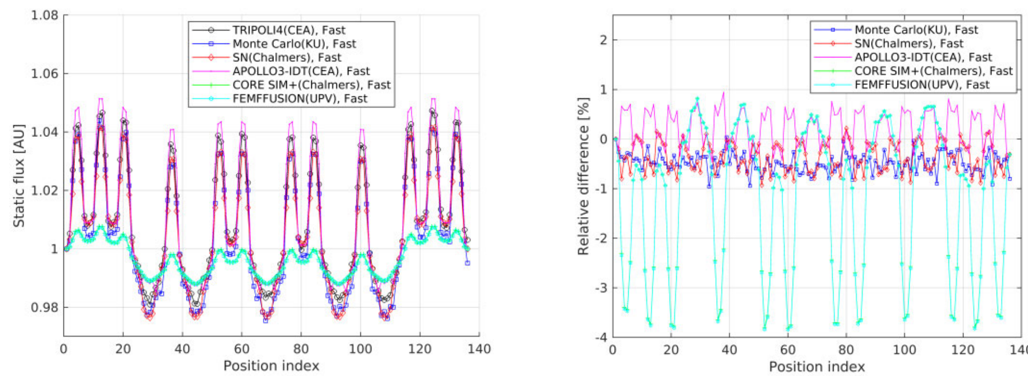

Figure 2. Fast static flux (left) and relative differences with respect to TRIPOLI-4® (right), along the main diagonal of the fuel assembly crossing the perturbed fuel pin

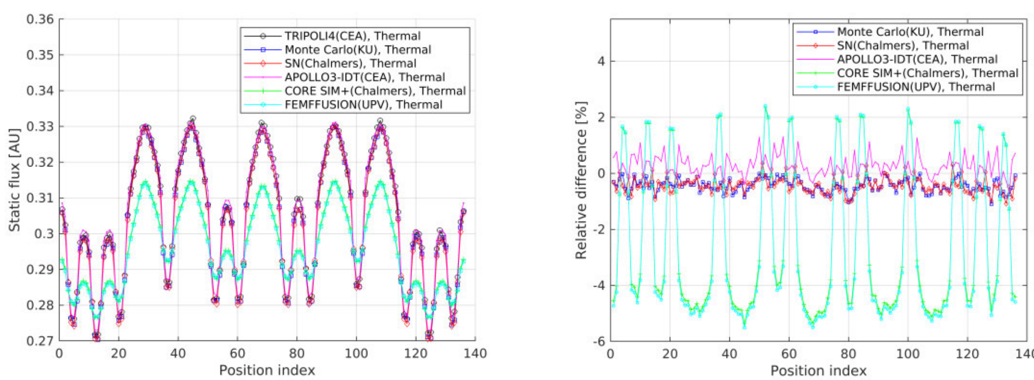

Figure 3. Thermal static flux (left) and relative differences with respect to TRIPOLI-4® (right), along the main diagonal of the fuel assembly crossing the perturbed fuel pin

For the neutron noise calculated in the two exercises discussed in section 2, the Monte Carlo solver developed by Kyoto University, the IDT solver embedded in APOLLO3®, and the discrete ordinates solver developed by Chalmers are in good agreement with TRIPOLI-4®. The differences estimated with respect to TRIPOLI-4® are between $\pm 2\%$ for the relative noise amplitude of the fast group (see Figure 4 for exercise 1 and Figure 5 for exercise 2) and of the thermal group (see Figures 6 for exercise 1 and Figure 7 for exercise 2).

For the diffusion-based solvers, the largest discrepancies are found close to the neutron noise source, where the diffusion approximation is expected to be less reliable. In exercise 1, the maximum relative differences between CORE SIM+ and TRIPOLI-4® are about -1.4% for the relative fast noise amplitude (see Figure 4) and -9% for the relative thermal noise amplitude (see Figure 6). In exercise 2, the biggest relative differences between CORE SIM+ and TRIPOLI-4® are about -3% for the relative fast noise amplitude (see Figure 5) and -8% for the relative thermal noise amplitude (see Figure 7). The solver FEMFFUSION was used only in the second exercise and its relative differences with TRIPOLI-4® can reach about -2.5% in the fast group (see Figure 5) and about -13% in the thermal group (see Figure 7). Far from the noise source, the diffusion calculations are consistent with the results of the higher-order transport methods.

All the solvers predict very similar values for the phase of the noise. For example, the case of exercise 2 is considered; the fast noise phase is shown in Figure 8 and the thermal noise phase in Figure 9. The relative differences with respect to TRIPOLI-4® are between $\pm 0.15\%$ for all the frequency-domain solvers. FEMFFUSION gives values with a slight shift (around -0.25%).

In the Monte Carlo simulations, uncertainties for the real and imaginary parts of the neutron noise are estimated separately via independent replicas. Because of the non-linear and highly correlated transformations required, a precise uncertainty for the amplitude and the phase cannot be assessed. However, in view of the high degree of statistical convergence on the real and imaginary parts, the results provided for the amplitude and phase are assumed to be sound and reliable.

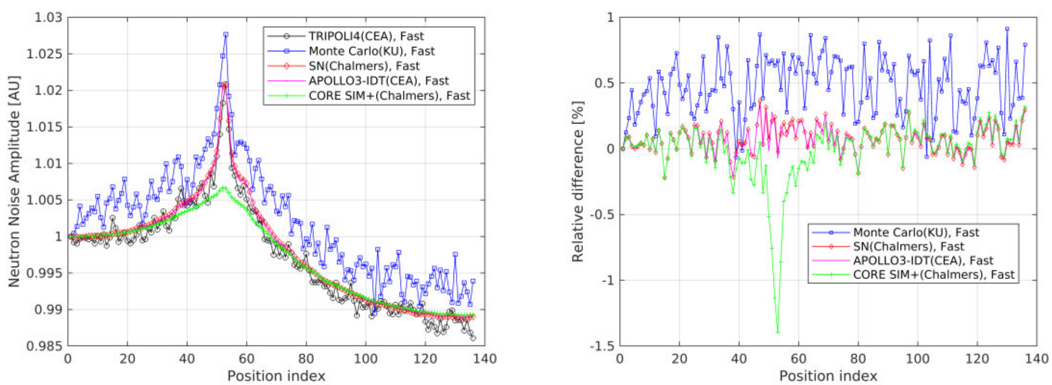

Figure 4. Exercise 1; relative fast noise amplitude (left) and relative differences with respect to TRIPOLI-4® (right), along the main diagonal of the fuel assembly crossing the perturbed fuel pin

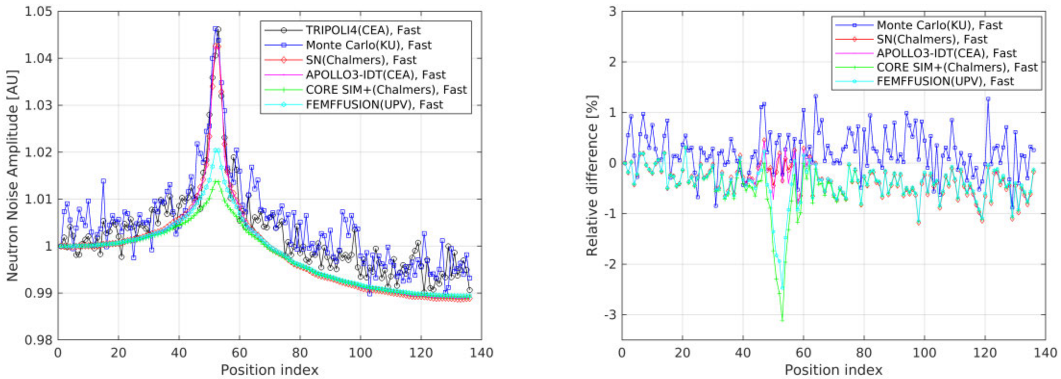

Figure 5. Exercise 2; relative fast noise amplitude (left) and relative differences with respect to TRIPOLI-4® (right), along the main diagonal of the fuel assembly crossing the perturbed fuel pin

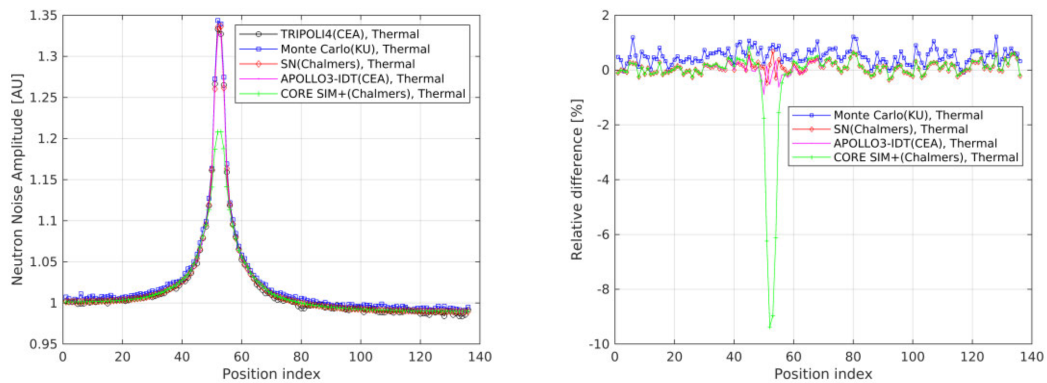

Figure 6. Exercise 1; relative thermal noise amplitude (left) and relative differences with respect to TRIPOLI-4® (right), along the main diagonal of the fuel assembly crossing the perturbed fuel pin

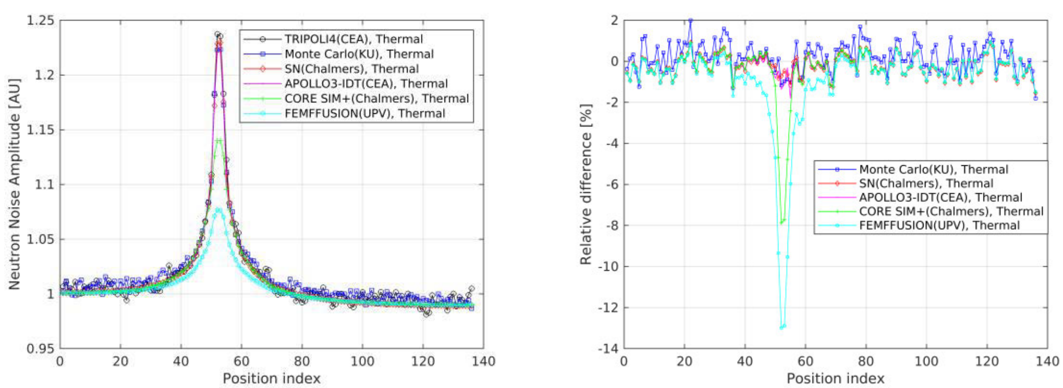

Figure 7. Exercise 2; relative thermal noise amplitude (left) and relative differences with respect to TRIPOLI-4® (right), along the main diagonal of the fuel assembly crossing the perturbed fuel pin

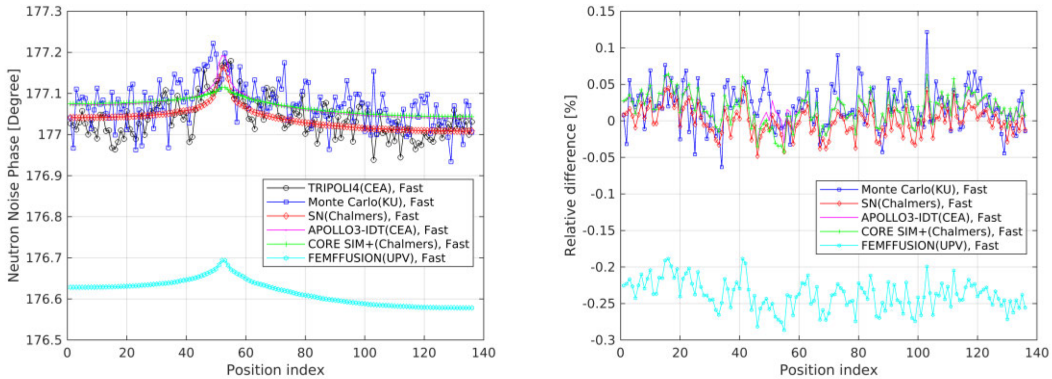

Figure 8. Exercise 2; fast noise phase (left) and relative differences with respect to TRIPOLI-4® (right), along the main diagonal of the fuel assembly crossing the perturbed fuel pin

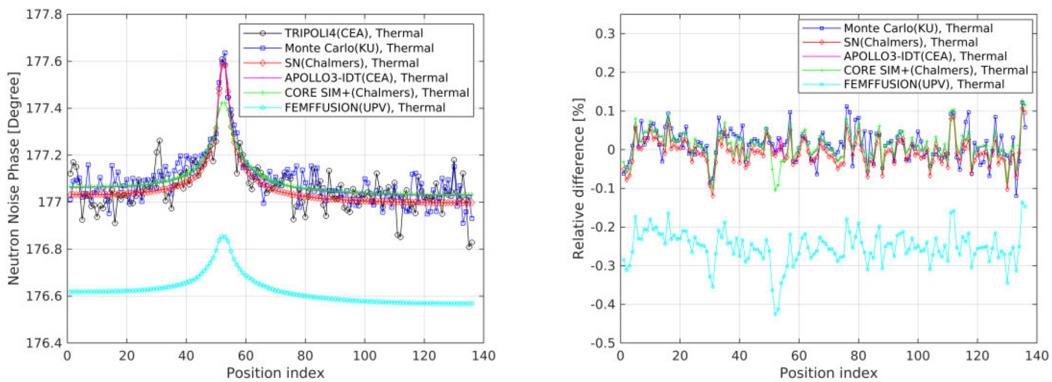

Figure 9. Exercise 2; thermal noise phase (left) and relative differences with respect to TRIPOLI-4® (right), along the main diagonal of the fuel assembly crossing the perturbed fuel pin

5. CONCLUSIONS

Several solvers are being developed and applied to analyze neutron noise problems within the CORTEX project. In this work, two numerical exercises based on a 2-D simplified UOX fuel assembly for PWRs are used to compare the solvers and explore their features. The higher-order deterministic neutron transport methods provide consistent results with respect to the Monte Carlo solvers. The neutron noise calculated using diffusion-based solvers shows discrepancies that can be significant, in particular for the thermal amplitude of the neutron noise in the region close to the neutron noise source. All the solvers predict very similar values for the noise phase.

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