UNDERSTANDING THE NEUTRON NOISE INDUCED BY FUEL ASSEMBLY VIBRATIONS IN LINEAR THEORY

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ABSTRACT

This paper investigates the effects of stationary vibrations of fuel assemblies in nuclear reactors. The incentive of this work is to understand the spatial pattern of the induced stationary fluctuations of the neutron flux. For that purpose, an as simple as possible system was considered: a one-dimensional three-region system, for which the boundaries between the regions are vibrating. Using one-group diffusion theory, it is demonstrated that in the frequency domain and in linear theory the amplitude and phase of the fluctuations in neutron flux exhibit different patterns, stemming from competing contributions between the point-kinetic and the space-dependent responses associated to each moving boundary. The results of this work can be used for interpreting the results of more sophisticated tools used for modelling the effect of fuel assembly vibrations onto the neutron flux, as well as measurements in reactors where vibrations of fuel assemblies occur.

KEYWORDS: neutron noise, fuel assembly vibrations, diffusion theory, linear theory

1. INTRODUCTION

A special emphasis of the European Horizon 2020 CORTEX project [1] (CORe monitoring Techniques and EXperimental validation and demonstration) lies on the modelling of the effects on the neutron flux of stationary vibrations of fuel assemblies in nuclear reactors. For such a purpose, various tools and methods are being developed to properly model fuel assembly vibrations and their impact, with dedicated experiments performed at two research reactors: the CROCUS reactor at the Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland and the AKR-2 reactor at the Technical University of Dresden, Dresden, Germany. The validation efforts first concentrated on the former facility, with various measurement campaigns performed where the stationary movement of 18 fuel rods took place in a controlled manner at a given angular frequency ω_0 [2]. Validation on the latter is on-going, with the analysis of various measurements corresponding to either an absorber of variable strength or a vibrating absorber.

In view of applying the developed numerical solvers to the interpretation of the experimental measurements, extensive code-to-code comparisons were carried out. It was noticed that the spatial dependence of the amplitude and phase of the induced neutron noise as computed by the solvers based on the linear theory had different patterns depending on the system configuration, with the neutron noise formally defined as the deviation of the neutron flux from its time-averaged value. Moreover, although the fuel assemblies are vibrating at the fundamental angular frequency ω_0, higher harmonics in the response of the system are excited. In some conditions and when the amplitude of the vibration becomes comparable to the radius of the fuel pins, the amplitude of the $2\omega_0$ harmonics predicted by the solvers was found to be larger than the one of the ω_0 harmonics [3].

The purpose of this paper is to highlight, via numerical simulations, the physical mechanisms responsible for the observed structure of the induced neutron noise, both at ω_0 and at $2\omega_0$. After introducing the representation of fuel assembly vibrations in neutron kinetic codes in Section 2, emphasis is put in Section 3 on modelling the corresponding effect on the neutron noise in the simplest manner using one-group diffusion theory. Some conclusions are drawn in Section 4.

2. MODELLING OF THE NOISE SOURCE IN CASE OF FUEL ASSEMBLY VIBRATIONS

Fuel assembly vibrations along a given direction x can be seen as the displacement $\varepsilon(t)$ of structures from their equilibrium positions. From a neutronic viewpoint, the structures and their surroundings are represented by sets of homogeneous regions, each having defined macroscopic cross-sections. Assuming that the system is infinite in the directions perpendicular to the direction of vibrations, the physics of vibrating structures can be studied by first considering two adjacent homogeneous regions I and II, as depicted in Fig. 1. Vibrations would then correspond to the displacement $\varepsilon(t)$ of the instantaneous boundary $b(t)$ from its equilibrium position b_0. The various pins constituting a fuel assembly could then be easily modelled by stacking several copies of the elementary model represented in Fig. 1. Furthermore, in a linear treatment of the neutron noise, as adopted in this work, the neutron noise induced by multiple noise sources is simply given by the superposition of the individual responses to each noise source. This thus justifies the emphasis on such an elementary lattice.

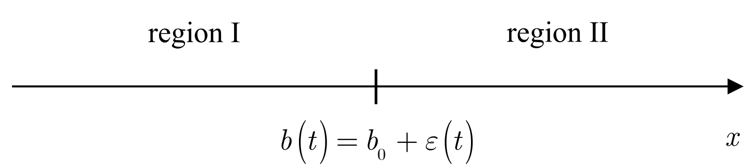

Figure 1. Elementary one-dimensional lattice representing two homogeneous regions and their time-dependent boundary.

The spatial variation of any static macroscopic cross-section $\Sigma_{\alpha,0}$ of type α can thus be represented as:

$$\Sigma_{\alpha,0}(x) = \left[1 - \mathrm{H}(x - b_0)\right]\Sigma_{\alpha,I} + \mathrm{H}(x - b_0)\Sigma_{\alpha,II} \quad (1)$$

where H is the Heaviside function, and $\Sigma_{\alpha,I}$ and $\Sigma_{\alpha,II}$ represent the (static) macroscopic cross-section of type α for regions I and II, respectively. When the position of the boundary is perturbed as

$b(t) = b_0 + \varepsilon(t)$, the deviation of the macroscopic cross-section of type α with respect to its static value $\Sigma_{\alpha,0}(x)$ is given as:

$$\delta\Sigma_\alpha(x,t) = -\Delta\Sigma_\alpha \mathrm{H}(x - b_0) + \Delta\Sigma_\alpha \mathrm{H}(x - b_0 - \varepsilon(t)) \tag{2}$$

with $\Delta\Sigma_\alpha = \Sigma_{\alpha,II} - \Sigma_{\alpha,I}$. Assuming a sinusoidal displacement of the position of the boundary from its equilibrium position as $\varepsilon(t) = d\sin(\omega_0 t)$, adopting the model of Rouchon and Sanchez [4], one could demonstrate that in the frequency domain we have:

- If $x \in [b_0 - d; b_0 + d]$:

$$\begin{aligned}\delta\Sigma_\alpha(x,\omega) &= -\mathrm{sign}[\tau(x)] \times \Delta\Sigma_\alpha \times [\pi - 2\omega_0 |\tau(x)|] \times \delta(\omega) \\ &+ \sum_{k=-\infty}^{+\infty} \frac{2i\Delta\Sigma_\alpha}{2k+1} \cos[(2k+1)\omega_0\tau(x)] \times \delta[\omega - (2k+1)\omega_0] + \sum_{\substack{k=-\infty \\ k\neq 0}}^{+\infty} \frac{2\Delta\Sigma_\alpha}{2k} \sin[2k\omega_0\tau(x)] \times \delta[\omega - 2k\omega_0]\end{aligned} \tag{3}$$

- If $x \notin [b_0 - d; b_0 + d]$:

$$\delta\Sigma_\alpha(x,\omega) = 0 \tag{4}$$

with

$$\tau(x) = \frac{1}{\omega_0} \arcsin\left(\frac{x - b_0}{d}\right) \tag{5}$$

One notices from the above that, in addition to a perturbation at the fundamental angular frequency ω_0, the movement of the boundary also creates perturbations at higher harmonics $n\omega_0$, with $n \geq 2$.

3. MODELLING OF THE INDUCED NEUTRON NOISE

In the following, the neutron noise induced by fuel assembly vibrations is considered in one-group diffusion theory, using the linearization of the noise equations, in the frequency domain and without any thermal-hydraulic feedback. To get physical insight, the system is kept as simple as possible, i.e. we choose to model a three-region system of size $2a$ with $a = 1.5$ m. The inner region is 1 cm wide. The boundaries between adjacent regions are moving according to the model presented in Section 2 in order to mimic the effect of vibrating structures, with $d =$ 0.5 cm and $\omega_0/2\pi =$ 1 Hz. The system can thus be considered as representative of a fuel pin vibrating in two neighboring homogeneous regions. The macroscopic cross-sections considered in this work are given in Table I.

The calculations are performed both semi-analytically and numerically using a modified version of the CORE SIM tool [5]. The modifications introduced allow modelling a one-dimensional system with a large number of nodes, making it possible to describe the spatial distribution of the noise source presented in Section 2 on sub-millimeter scales, i.e. the node size is 0.01 cm. Furthermore, zero flux boundary conditions were used. The incentive of complementing numerical simulations with semi-analytical simulations is to make sure that the CORE SIM tool is able to properly reproduce the induced neutron noise on such small scales.

For the case of three consecutive regions labelled I, II and III, respectively (with b and c representing the position of the corresponding interfaces), the semi-analytical solution is formally given in the frequency domain as:

$$\delta\phi\left(x,\omega\right)=\int G\left(x,x',\omega\right)\delta S\left(x',\omega\right)\phi_0\left(x'\right)dx' \tag{6}$$

where the Green's function $G\left(x,x',\omega\right)$ can be expressed as:

- If $x'\leq b$:

$$G\left(x,x',\omega\right)=\begin{cases} C_I\sin\left[B_I\left(\omega\right)\left(x+a\right)\right] \text{ for } x\leq x' \\ D_I\exp\left[iB_I\left(\omega\right)x\right]+E_I\exp\left[-iB_I\left(\omega\right)x\right] \text{ for } x'\leq x\leq b \\ F_{II}\exp\left[iB_{II}\left(\omega\right)x\right]+\mathrm{G}_{II}\exp\left[-iB_{II}\left(\omega\right)x\right] \text{ for } b\leq x\leq c \\ H_{III}\sin\left[B_{III}\left(\omega\right)\left(x-a\right)\right] \text{ for } c\leq x \end{cases} \tag{7}$$

- If $b\leq x'\leq c$:

$$G\left(x,x',\omega\right)=\begin{cases} C_I\sin\left[B_I\left(\omega\right)\left(x+a\right)\right] \text{ for } x\leq b \\ D_{II}\exp\left[iB_{II}\left(\omega\right)x\right]+E_{II}\exp\left[-iB_{II}\left(\omega\right)x\right] \text{ for } b\leq x\leq x' \\ F_{II}\exp\left[iB_{II}\left(\omega\right)x\right]+G_{II}\exp\left[-iB_{II}\left(\omega\right)x\right] \text{ for } x'\leq x\leq c \\ H_{III}\sin\left[B_{III}\left(\omega\right)\left(x-a\right)\right] \text{ for } c\leq x \end{cases} \tag{8}$$

- If $x'\geq c$:

$$G\left(x,x',\omega\right)=\begin{cases} C_I\sin\left[B_I\left(\omega\right)\left(x+a\right)\right] \text{ for } x\leq b \\ D_{II}\exp\left[iB_{II}\left(\omega\right)x\right]+E_{II}\exp\left[-iB_{II}\left(\omega\right)x\right] \text{ for } b\leq x\leq c \\ F_{III}\exp\left[iB_{III}\left(\omega\right)x\right]+G_{III}\exp\left[-iB_{III}\left(\omega\right)x\right] \text{ for } c\leq x\leq x' \\ H_{III}\sin\left[B_{III}\left(\omega\right)\left(x-a\right)\right] \text{ for } x'\leq x \end{cases} \tag{9}$$

Table I: Macroscopic cross-section data used in the reactor model.

	Inner region	Outer regions
Diffusion coefficient, D [cm]	1.3410	1.3410
Macroscopic absorption cross-section, Σ_a [cm^{-1}]	1.8983x10^{-2}	2.0881x10^{-2}
Macroscopic fission cross-section times the average number of neutrons emitted per fission event, $\upsilon\Sigma_f$ [cm^{-1}]	2.3300x10^{-2}	2.0970x10^{-2}

In Eq. (6), $\phi_0\left(x\right)$ represents the static flux corresponding to the three-region system (for which the solution is not detailed here for the sake of brevity), whereas the noise source $\delta S\left(x,\omega\right)$ is defined as:

$$\delta S\left(x,\omega\right)=\delta\Sigma_a\left(x,\omega\right)-\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right)\frac{\delta\upsilon\Sigma_f\left(x,\omega\right)}{k_{eff}} \tag{10}$$

where k_{eff} is the effective multiplication factor of the unperturbed system. The perturbations of the macroscopic cross-sections $\delta\Sigma_a\left(x,\omega\right)$ and $\delta\upsilon\Sigma_f\left(x,\omega\right)$ are given by Eqs. (3) and (4), applied to each of the boundaries located at b and c. The spatial dependence of the amplitude and phase of the noise source δS defined in Eq. (10) is represented in Fig. 2 for a large reactor with the set of cross-sections defined in Table I, both at the fundamental frequency of $\omega_0/2\pi=1$ Hz and at the $\omega_0/\pi=2$ Hz harmonics. In all equations above, the notations are standard and the "dynamic" buckling constants are defined as:

$$B_R^2\left(\omega\right)=\frac{\left(1-\dfrac{i\omega\beta}{i\omega+\lambda}\right)\dfrac{\upsilon\Sigma_{f,R}}{k_{eff}}-\Sigma_{a,R}-\dfrac{i\omega}{v}}{D_R} \tag{11}$$

with R representing the region label (i.e. $R\equiv$ I or II or III). The various constants appearing in Eqs. (7)-(9), i.e. C_I, $D_{I/II}$, $E_{I/II}$, $F_{II/III}$, $G_{II/III}$ and H_{III} are obtained by expressing:

- The continuity of the Green's function at the noise source location x' and in b and c.
- The continuity of the gradient of the Green's function in b and c, and the discontinuity of the gradient of the Green's function at the noise source location x'.

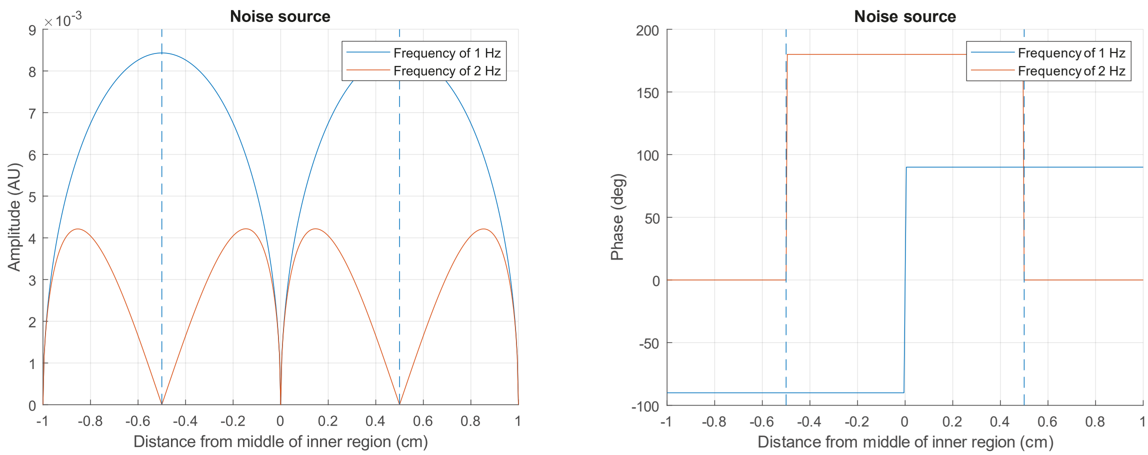

Figure 2. Amplitude (left figure) and phase (right figure) of the noise source at 1 Hz and 2 Hz in the case of an inner region moving in relation to two outer regions in a large reactor. The plots correspond to the case of a centrally located innermost region (the plots are nevertheless identical to the naked eye for an off-centered innermost region).

The comparison between the CORE SIM simulations and the semi-analytical solution obtained by Eq. (6) is represented for $b=$ -35.5 cm and $c=$ -34.5 cm in Figs. 3 and 4, giving the spatial dependence of the solution throughout the entire system and around the perturbed boundaries, respectively, for the considered system and set of macroscopic cross-sections. Comparisons are carried out at both the fundamental frequency of $\omega_0/2\pi=$ 1 Hz and at the $\omega_0/\pi=$ 2 Hz harmonics. As seen in those figures, the agreement between the CORE SIM and semi-analytical solutions is excellent, both regarding the amplitude and phase of the induced neutron noise. This demonstrates the capability of CORE SIM to correctly estimate the neutron noise for closely located moving boundaries.

In the following of this paper, two cases are then investigated in further detail:

- The case of a centrally located innermost region, for which $b=$ -0.5 cm and $c=$ +0.5 cm.
- The case of an off-centered innermost region, for which $b=$ -35.5 cm and $c=$ -34.5 cm (this case corresponds to the system configuration used for verifying the CORE SIM solution presented above).

In both cases, the amplitude and phase of the induced neutron noise for both the fundamental frequency of $\omega_0/2\pi=$ 1 Hz and of the higher harmonics at $\omega_0/\pi=$ 2 Hz were estimated. In order to get better insight, the point-kinetic component of the induced neutron noise and its deviation from point-kinetics were also determined. The results for the centrally located innermost region are represented in Fig. 5, whereas the ones for the off-centered innermost region are represented in Fig. 6.

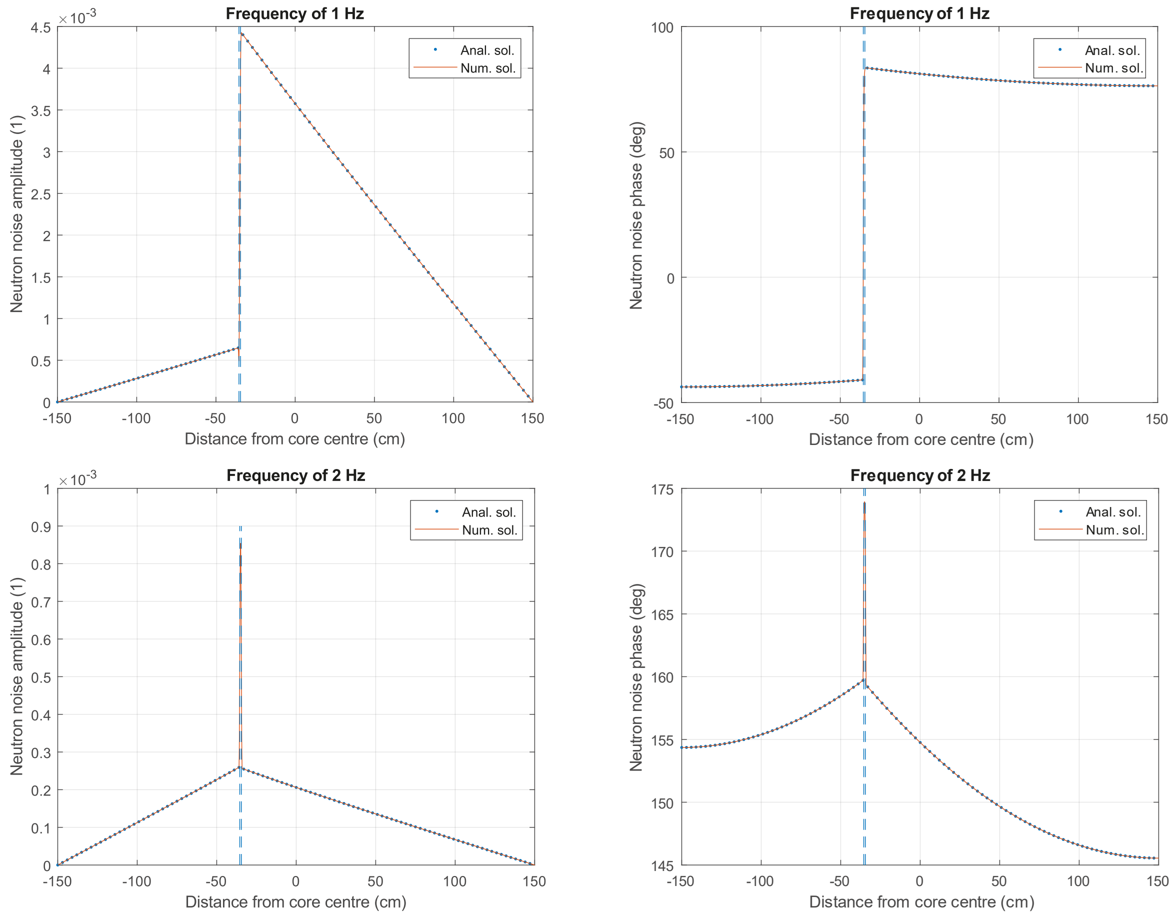

Figure 3. Comparisons between the results of the CORE SIM solution (labelled "Num. sol.") and the semi-analytical solution (labelled "Anal. Sol.) throughout the entire system. The amplitude of the induced neutron noise is given on the left figures and its phase on the right figures, whereas the results at 1 Hz are given in the top figures and the ones at 2 Hz are given in the bottom figures.

For the 1 Hz component, the noise source is represented by an anti-symmetrical (out-of-phase) perturbation around the center of the moving region. The noise source on the left-hand side of the inner region and the noise source on the right-hand side of the inner region each give a large point-kinetic response, thus being out-of-phase between each other. For the 2 Hz component, the noise source on the left-hand side of the inner region and the noise source on the right-hand side of the inner region are both anti-symmetrical (out-of-phase) perturbations around the respective perturbed boundaries, as seen in Fig. 2. Nevertheless, the total perturbation with respect to the two perturbed boundaries is symmetrical around the center of the moving region, thus leading to point-kinetic responses associated to each noise source being in-phase. Since each perturbation is anti-symmetrical around each perturbed boundary, the point-kinetic response of each perturbation is relatively small. As a result, each perturbation gives rise to significant deviations from point-kinetics, with an overwhelming response on the side of the system being perturbed.

Based on the simulations of the induced neutron noise, the following conclusions can be drawn in the case of perfectly symmetrical perturbations (i.e. perturbation in the middle of the system – see Fig. 5):

- The component at the fundamental frequency is very small, since the contributions from each of the corresponding noise sources are anti-symmetrical (out-of-phase) and their effect cancel each other. In such a case, the resulting induced neutron noise is essentially the deviation from point-kinetics of the contribution of each noise source and is thus very small. The ratio between the effect of each noise source and the combined effect is very large.

- The component at two times the fundamental frequency, on the other hand, is the result of two symmetrical (in-phase) perturbations. In the present configuration, the very small flux gradient at the origin of the system introduces a negligible reactivity effect, and thus a small point-kinetic response associated to each noise source. The individual responses are anti-symmetrical with respect to each of the perturbed boundaries, thus giving a non-overwhelming point-kinetic response for each perturbed boundary. Nevertheless, due to the symmetrical (in-phase) nature of the two perturbations, the combination of the effect of the two noise sources cancel the respective deviations from point-kinetics due to each of the two noise sources, thus re-establishing an apparent point-kinetic behavior for the total noise.

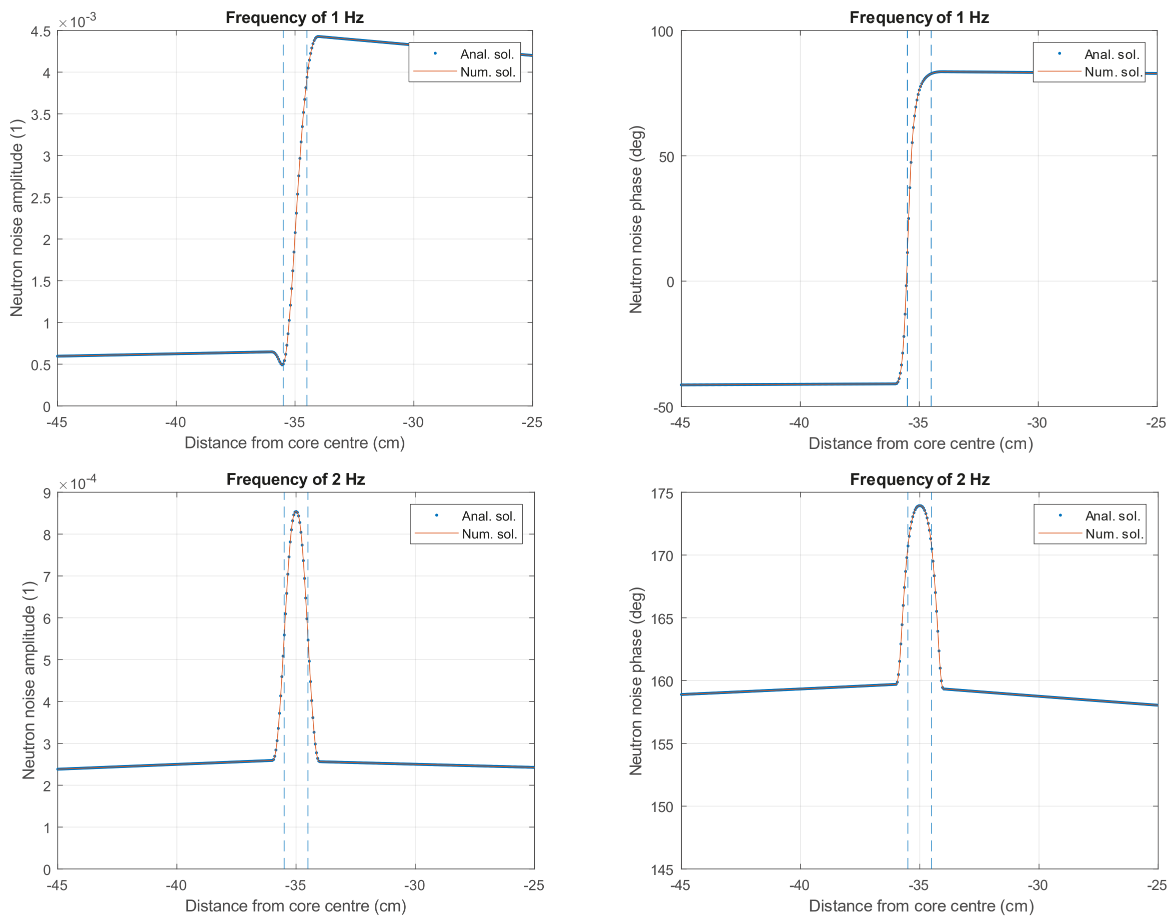

Figure 4. Comparisons between the results of the CORE SIM solution (labelled "Num. sol.") and the semi-analytical solution (labelled "Anal. Sol.) in the close vicinity of the moving boundaries. The amplitude of the induced neutron noise is given on the left figures and its phase on the right figures, whereas the results at 1 Hz are given in the top figures and the ones at 2 Hz are given in the bottom figures.

In the case of non-symmetrical perturbations (i.e. off-centered perturbations – see Fig. 6):

- For the component at the fundamental frequency, there is a slight asymmetry in the two noise sources, leading to a non-perfect compensation of the individual point-kinetic responses. As a result, the overall response contains a significant point-kinetic component, as compared to the case of symmetrical perturbations. Interference effects between the phase of the point-kinetic component and the deviation from point-kinetics give a rather complex structure for the resulting induced neutron noise.

- For the component at two times the fundamental frequency, because of the larger static flux gradient, each noise source gives essentially a point-kinetic response, being out-of-phase between each other – the absence of perfect anti-symmetry between the two noise sources gives two reactivity effects being out-of-phase. As a result, the combination of the effect of the two noise sources results in the subtraction of essentially two point-kinetic responses. This means that the resulting induced neutron noise contains a significant point-kinetic contribution.

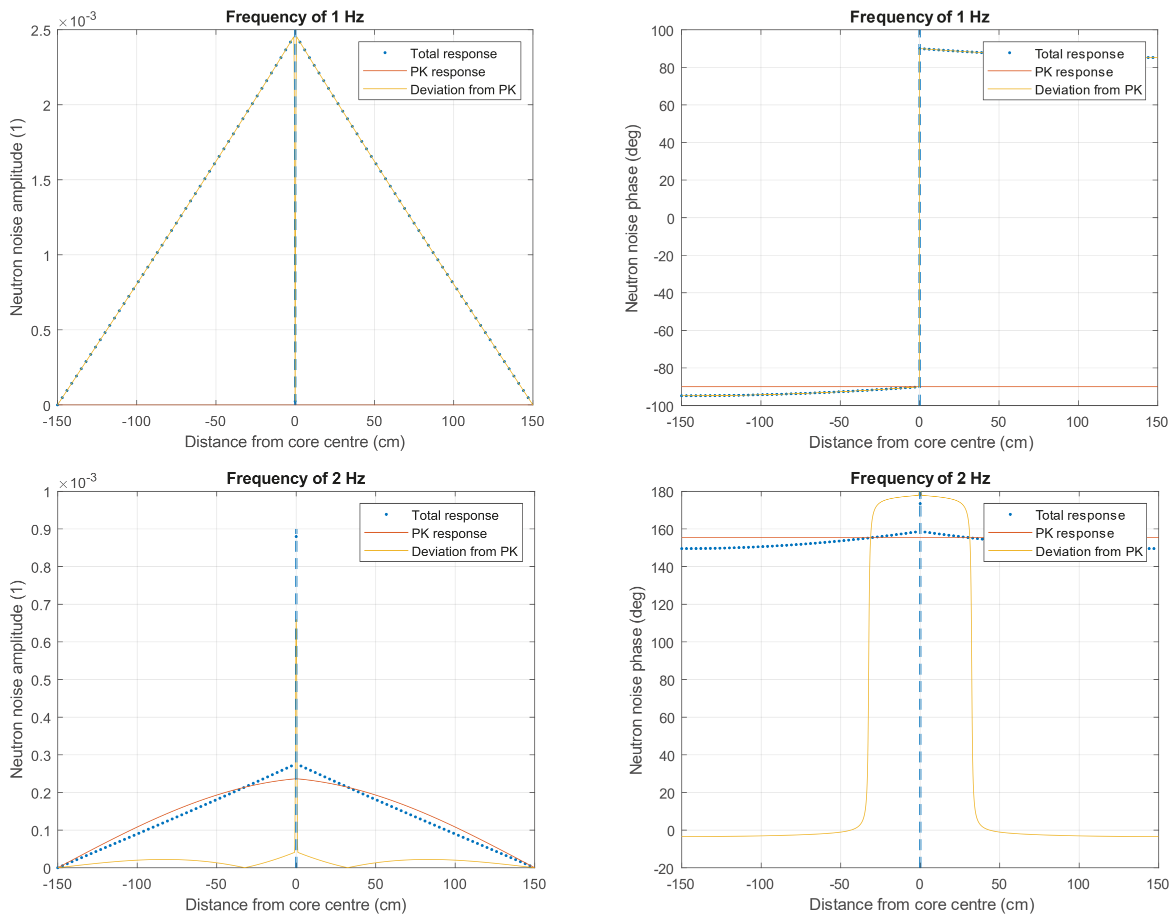

Figure 5. Amplitude (left figures) and phase (right figures) of the induced neutron noise at 1 Hz (top figures) and 2 Hz (bottom figures) for a central moving region in one-group diffusion theory – the induced neutron noise is represented in blue, its point-kinetic component in red and its deviation from point-kinetics in yellow.

Although using diffusion theory might be questionable on small scales in the close vicinity of the perturbed boundaries, additional calculations carried out in small systems (not reported here for the sake of brevity) using transport theory and diffusion theory demonstrated that the same structure of the induced neutron noise is obtained in both formalisms. Moreover, both formalisms also showed that the component at two times the fundamental frequency dominates in amplitude for central perturbations in case of large static flux gradients. This "explosion" of the component at two times the fundamental frequency was nevertheless explained to be an artefact of linear theory, i.e. using the non-linear neutron noise equations does not give a component at two times the fundamental frequency being larger than the one at the fundamental frequency [6].

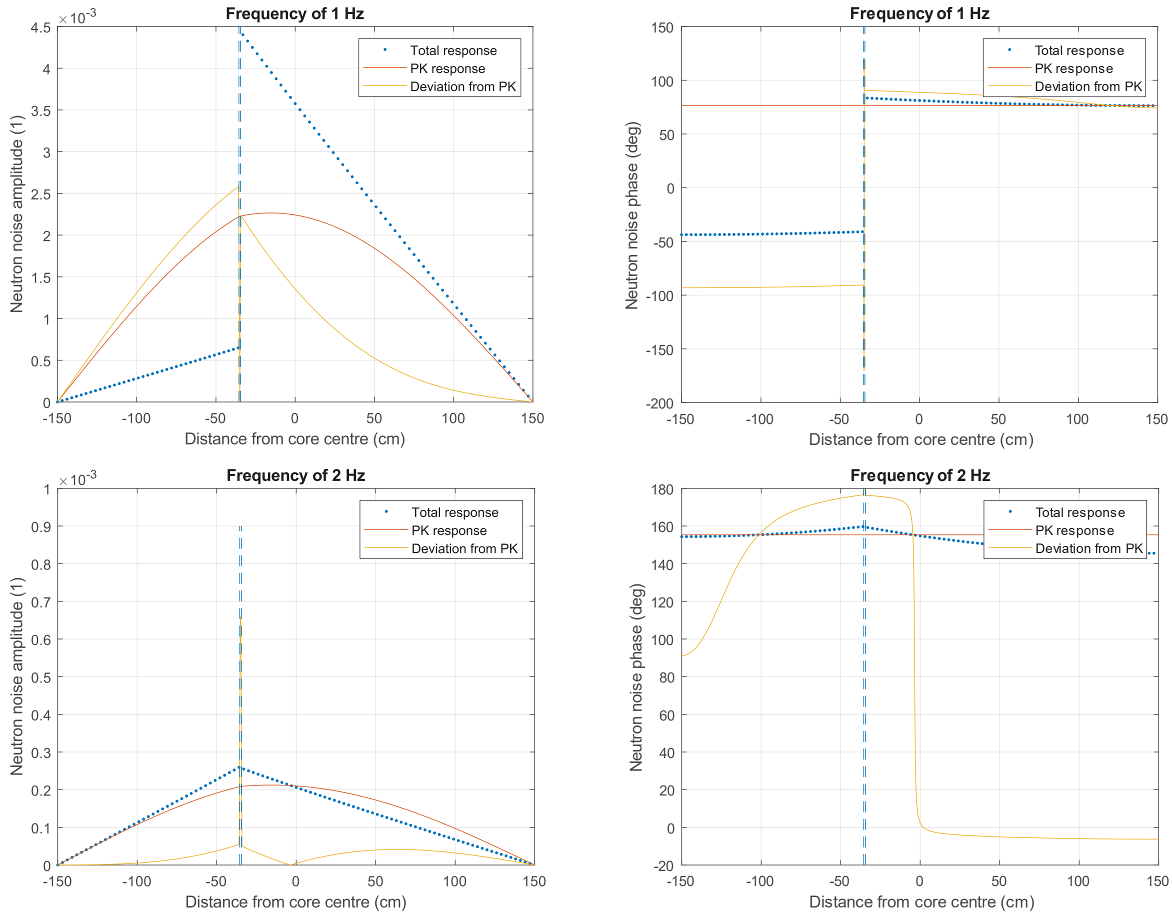

Figure 6. Amplitude (left figures) and phase (right figures) of the induced neutron noise at 1 Hz (top figures) and 2 Hz (bottom figures) for a peripheral moving region in one-group diffusion theory – the induced neutron noise is represented in blue, its point-kinetic component in red and its deviation from point-kinetics in yellow.

4. CONCLUSIONS

As demonstrated in this paper, the vibrations of fuel assemblies lead to an intricate combination of point-kinetic responses and space-dependent responses associated to each of the moving boundaries. Depending on the position of the moving regions, the harmonics being considered, the existing gradient of the static flux, rather different responses for the total noise are observed in linear theory. This paper, via dedicated simulations, shed light on a better understanding of the underlying physical mechanisms leading to such a variety of system responses. Despite the simplicity of the model, additional simulations performed in two-group theory for a one-dimensional heterogeneous representation of a commercial pressurized water reactor (modelling all fuel pins explicitly) demonstrate that the same qualitative behavior of the induced neutron noise can be observed, as shown in Fig. 7. In those simulations, one fuel pin, located around the abscissa $x \approx$-106 cm from the center of a core of size 3.61 m, is vibrating. With the present set of cross-sections for this system, only the phase of the induced neutron noise gives a somewhat milder dependence as compared to the results of the three-region system in one-group diffusion theory. Further studies will investigate the vibrations of several consecutive fuel pins, for which interference effects between the pins are expected. It should nevertheless be emphasized that, in linear theory, the neutron noise induced by several fuel pins is given by the superposition of the neutron noise induced by each vibrating fuel pin. This further highlights the benefits of better understanding the neutron noise structure from this work.

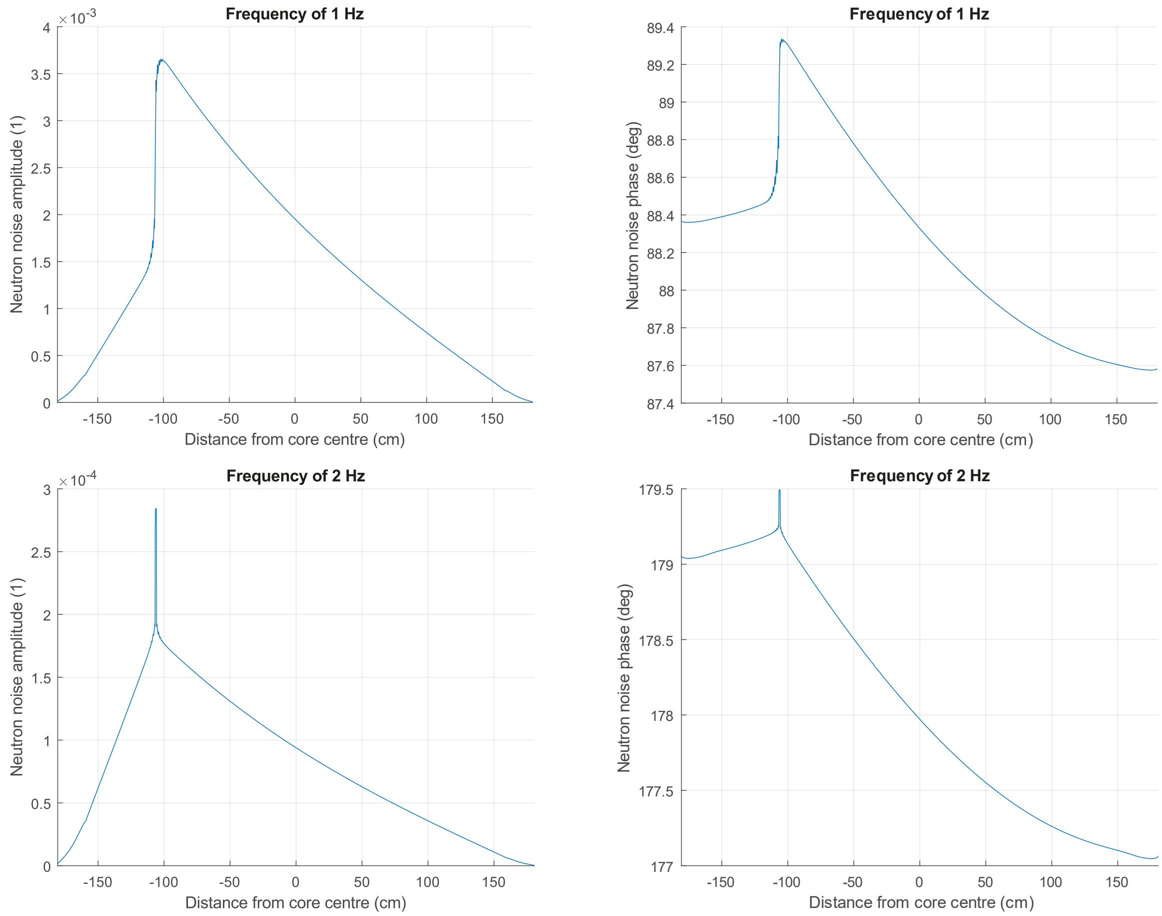

Figure 7. Amplitude (left figures) and phase (right figures) of the induced neutron noise at 1 Hz (top figures) and 2 Hz (bottom figures) for a peripheral moving region in two-group diffusion theory.

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