CORTEX

Core monitoring techniques and experimental validation and demonstration

Power reactor noise

CORTEX joint WP1-WP2 Workshop

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Introduction

- Different communities use different “names”/“labels” in power reactor noise, leading to some confusion.
- In terms of modelling, many different approaches exist, all having advantages/disadvantages.
- Although the modelling of the induced neutron noise is essential, the modelling of the noise source is equally important.

Introduction

- Presentation thus focusing on:
  - Some theoretical remarks on power reactor noise
  - An overview of the various approaches for modelling the induced neutron noise
  - Noise source modelling

Some theoretical remarks on power reactor noise

Some theoretical remarks on power reactor noise

- Notations:

  all time-dependent terms split into a mean value and a fluctuating part as

$$X(\mathbf{r},t) = X_0(\mathbf{r}) + dX(\mathbf{r},t)$$

  with

$$\left|dX(\mathbf{r},t)\right| = X_0(\mathbf{r}), \text{"}(\mathbf{r},t)$$

  and

$$\left\langle dX(\mathbf{r},t)\right\rangle = 0, \text{"}(\mathbf{r},t)$$

Some theoretical remarks on power reactor noise

- What is the point-kinetic component of the neutron noise?

Using the factorization:

$$\phi(\mathbf{r},t) = P(t)\times\psi(\mathbf{r},t)$$

with

$P(t)$ amplitude factor

$\psi(\mathbf{r},t)$ shape function

such that

$$\frac{\partial}{\partial t}\int \phi_0(\mathbf{r})\psi(\mathbf{r},t)d^3\mathbf{r} = 0$$

Some theoretical remarks on power reactor noise

- What is the point-kinetic component of the neutron noise?

  One obtains in first order:

$$df\left(\mathbf{r},t\right)=dP\left(t\right)f_0\left(\mathbf{r}\right)+dy\left(\mathbf{r},t\right)$$

  where one assumed:

$$P_0 = 1$$

$$y\left(\mathbf{r},t=0\right)=f_0\left(\mathbf{r}\right)$$

  - Point-kinetic response: $dP\left(t\right)f_0\left(\mathbf{r}\right)$
  - “Space-dependent” response: $dy\left(\mathbf{r},t\right)$

Some theoretical remarks on power reactor noise

- What is the point-kinetic component of the neutron noise?

The fluctuations of the amplitude factor are further given, in the frequency domain, as:

$$\delta P(\omega) = G_0(\omega)\,\delta\rho(\omega)$$

with

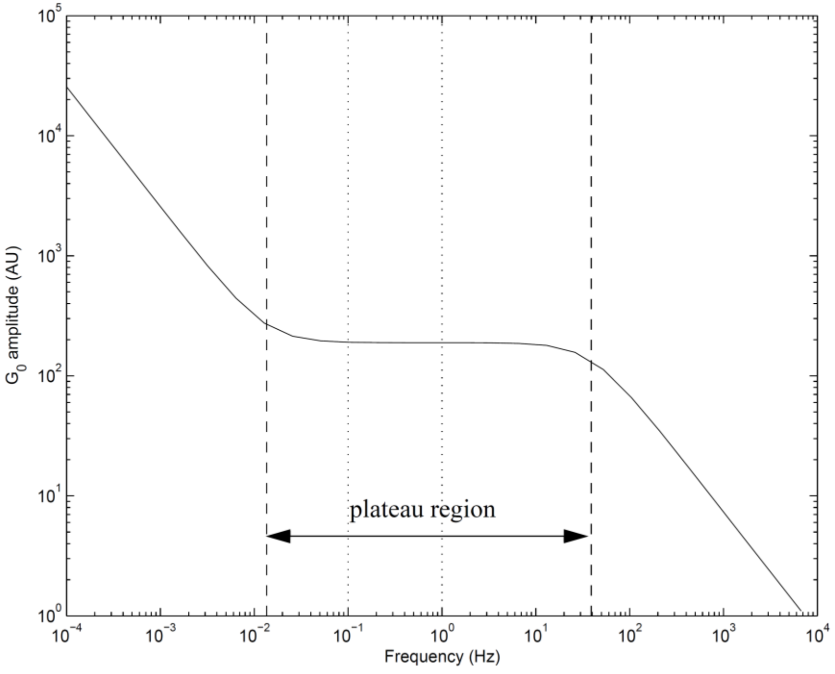
$$G_0(\omega) = \frac{1}{i\omega\left(\Lambda_0 + \dfrac{\beta}{i\omega + \lambda}\right)}$$

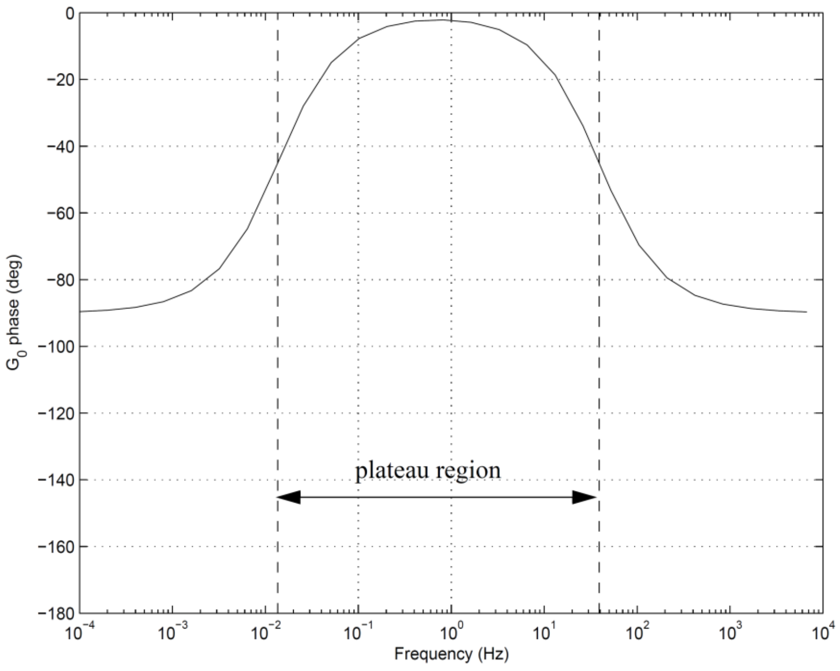
zero-power reactor transfer function

(better name: *point-kinetic* zero-power reactor transfer function)

Some theoretical remarks on power reactor noise

- What is the point-kinetic component of the neutron noise?





Amplitude and phase of the zero-power reactor transfer function

Some theoretical remarks on power reactor noise

- What is the point-kinetic component of the neutron noise?

  Remark:

  Even at zero power (i.e. without feedback), the reactor response deviates from point-kinetics in the most general case

Some theoretical remarks on power reactor noise

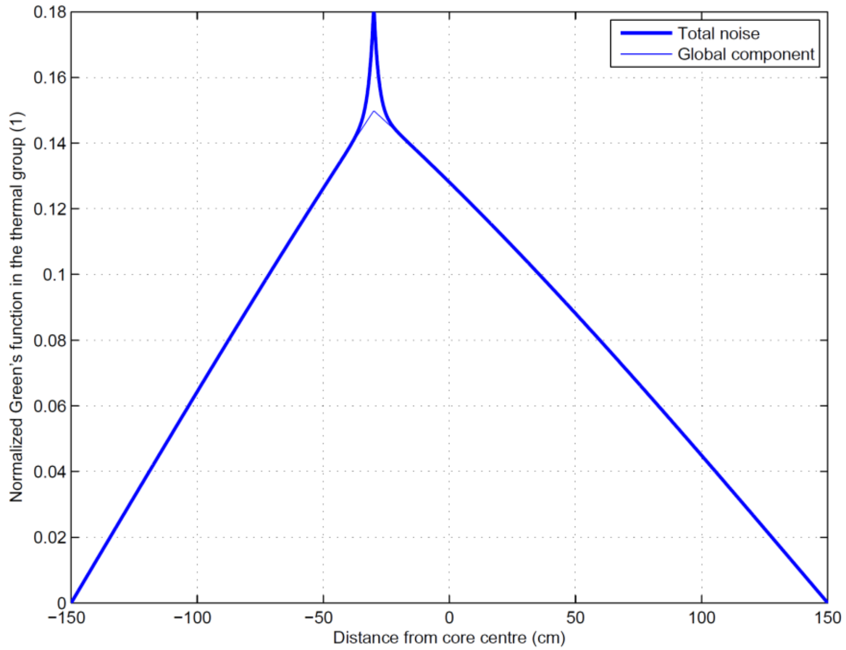
- What are the local and global components of the reactor noise?

  In more than or equal to two energy groups, the space-dependence of the induced neutron noise shows two relaxation lengths:

  - A short relaxation length: the local component
  - A long relaxation length: the global component

Some theoretical remarks on power reactor noise

- What are the local and global components of the reactor noise?



Example of the space-dependence of the amplitude of the thermal component of the Green's function in two-group diffusion theory (at 5 Hz)

Some theoretical remarks on power reactor noise

- What are the local and global components of the reactor noise?

Remarks:

- The local component does not exist in one-group theory!
- The global component should not be mistaken with the point-kinetic component!

Some theoretical remarks on power reactor noise

- What are the local and global components of the reactor noise?

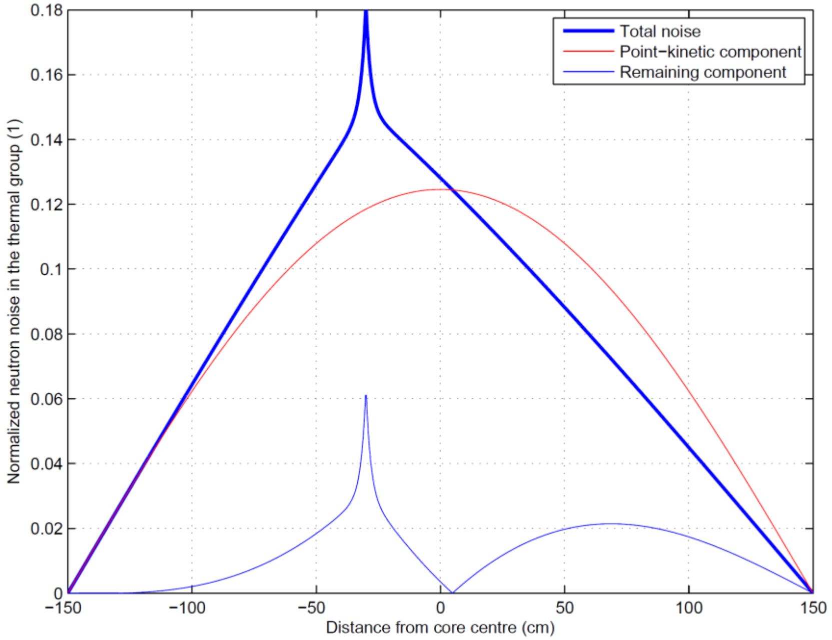


Illustration of the difference between the global component and the point-kinetic component (at 1 Hz)

Some theoretical remarks on power reactor noise

- Go to www.menti.com and use the code 60 79 61

Overview of the various approaches for modelling the induced neutron noise

Overview of the various approaches for modelling the induced neutron noise

- Once the noise source is modelled, need to estimate the response of the neutron flux to the applied perturbation

➢ Could be done using the neutron transport equation (Boltzmann equation):

$$\begin{aligned}
&\frac{1}{v(E)}\frac{\partial}{\partial t}\psi(\mathbf{r},\boldsymbol{\Omega},E,t)\\
&= -\boldsymbol{\Omega}\cdot\nabla\psi(\mathbf{r},\boldsymbol{\Omega},E,t) - \Sigma_t(\mathbf{r},E,t)\psi(\mathbf{r},\boldsymbol{\Omega},E,t)\\
&+ \int_{(4\pi)}\int_0^\infty \Sigma_s(\mathbf{r},\boldsymbol{\Omega}'\to\boldsymbol{\Omega},E'\to E,t)\psi(\mathbf{r},\boldsymbol{\Omega}',E',t)\,d^2\Omega'\,dE'\\
&+ \frac{1}{4\pi}\int_{-\infty}^{t}\int_0^\infty \nu(E')\Sigma_f(\mathbf{r},E',t')\phi(\mathbf{r},E',t')\left[(1-\beta)\chi^p(E)\delta(t-t') + \sum_{i=1}^{N_d}\chi_i^d(E)\lambda_i\beta_i e^{-\lambda_i(t-t')}\right]dt'\,dE'
\end{aligned}$$

Overview of the various approaches for modelling the induced neutron noise

- Neutron noise transport equation = integro-differential equations in the multi-dimensional phase space $(\mathbf{r},\mathbf{\Omega},E,t)$

➢ Simpler formalisms usually used for modelling nuclear reactor cores, such as the multi-group diffusion approximation:

$$\begin{aligned}\frac{1}{v_g}\frac{\partial \phi_g}{\partial t}(\mathbf{r},t) &= \nabla\cdot\left[D_g(\mathbf{r},t)\nabla\phi_g(\mathbf{r},t)\right] - \Sigma_{t,g}(\mathbf{r},t)\phi_g(\mathbf{r},t)\\ &+ \sum_{g'=1}^{G}\Sigma_{s0,g'\to g}(\mathbf{r},t)\phi_{g'}(\mathbf{r},t) + (1-\beta)\chi_g^p\sum_{g'=1}^{G}\nu_{g'}\Sigma_{f,g'}(\mathbf{r},t)\phi_{g'}(\mathbf{r},t) + \sum_{i=1}^{N_g}\lambda_i\chi_{i,g}^d C_i(\mathbf{r},t)\end{aligned}$$

with

$$\frac{\partial C_i}{\partial t}(\mathbf{r},t) = \beta_i\sum_{g'=1}^{G}\nu_{g'}\Sigma_{f,g'}(\mathbf{r},t)\phi_{g'}(\mathbf{r},t) - \lambda_i C_i(\mathbf{r},t), i = 1,\ldots,N_d$$

Overview of the various approaches for modelling the induced neutron noise

- Different approaches possible:
  - Time-domain modelling

    Advantages:
    - Existing time-domain codes could be used
    - Non-linear effects inherently accounted for
    - Thermal-hydraulic feedback automatically taken into account

    Disadvantages:
    - Lengthy calculations
    - Challenging to get a highly accurate solution for the noise
    - Codes originally not developed for that purpose
    - Lack of verification and validation for noise analyses

Overview of the various approaches for modelling the induced neutron noise

- Different approaches possible:
  - Frequency-domain modelling

    Time-domain equations transformed into frequency-domain equations according to the following procedure:

    - Splitting between mean values and fluctuations
    - Linear theory used because of the smallness of the fluctuations
    - Fourier-transform of the balance equations for the dynamical part only

Overview of the various approaches for modelling the induced neutron noise

- Different approaches possible:
  - Frequency-domain modelling

    Advantages:
    - Codes specifically developed for noise analysis, thus usually fully verified (validated?)
    - Highly accurate noise solution
    - Usually high flexibility in the modelling
    - Very fast calculations

    Disadvantages:
    - No commercial code available
    - Possible linear effects disregarded
    - Thermal-hydraulic feedback generally not taken into account (but could be)

Overview of the various approaches for modelling the induced neutron noise

- Codes used in CORTEX:

Code name	Domain		Non-linear terms		Angular resolution		Spatial resolution		Approach	
	Time	Frequency	Not modelled	Modelled	Diffusion	Transport	Fine-mesh	Coarse-mesh	Deterministic	Probabilistic
SIMULATE-3K	✓			✓	✓			✓	✓	
DYN3D	✓			✓		✓		✓	✓	
QUABBOX/ CUBBOX	✓			✓	✓			✓	✓	
PARCS	✓			✓	✓	(✓)		✓	✓	
FEMFUSSION	✓	✓		✓	✓		✓		✓	
APOLLO3®	✓			✓		✓	✓		✓	

Overview of the various approaches for modelling the induced neutron noise

- Codes used in CORTEX:

Code name	Domain		Non-linear terms		Angular resolution		Spatial resolution		Approach	
	Time	Frequency	Not modelled	Modelled	Diffusion	Transport	Fine-mesh	Coarse-mesh	Deterministic	Probabilistic
CORE SIM		✓	✓		✓			✓	✓	
CORE SIM+		✓	✓		✓		✓		✓	
Sn-based solver		✓	✓			✓	✓		✓	
Extension of MCNP		✓	✓			✓	✓			✓
Extension of TRIPOLI-4®		✓	✓			✓	✓			✓
Equivalence-based method using MCNP		✓	✓			✓		✓		✓

Noise source modelling

Noise source modelling

- Since all neutron transport codes use nuclear macroscopic cross-sections as input, need to convert "physical" perturbations into perturbations of macroscopic cross-sections.
- Perturbations can be defined:
  - In the time-domain, more or less as they are, with limitations/approximations due to the mesh used.
  - In the frequency-domain, after typically a first-order approximation of the perturbation, subsequently followed by a Fourier transform + limitations/approximations due to the mesh used.

➢ Modelling possibly supplemented by other modelling tools (e.g. fluid-structure modelling tool)

➢ Noise source modelling strongly dependent on the choices made by the user

Noise source modelling

- Different scenarios investigated in CORTEX:
  - “Absorber of variable strength”
  - “Vibrating absorber”
  - Axially-travelling perturbations
  - Inlet flow rate perturbations
  - Fuel assembly vibrations
  - Core barrel vibrations

Noise source modelling

- "Absorber of variable strength" type of noise source:
  - Localized perturbation of which its amplitude varies in time at a fixed position
  - Induced neutron noise given by the Green's function
  - The effect of all other types of noise source can be given using such Green's functions
  - Simplest type of perturbation but "academic" type of perturbation

NB: An inlet flow rate perturbation can be seen as an absorber of variable strength type of perturbation distributed along the height of the perturbed channel.

Noise source modelling

- “Vibrating absorber” type of noise source:
  - Lateral movement of the absorber represented as (weak absorber):

$$\delta\Sigma_{a,2}(\mathbf{r},t) = \gamma\theta(z - z_0)\left[\delta\left(\mathbf{r}_{xy} - \mathbf{r}_{p,xy} - \boldsymbol{\varepsilon}(t)\right) - \delta\left(\mathbf{r}_{xy} - \mathbf{r}_{p,xy}\right)\right]$$

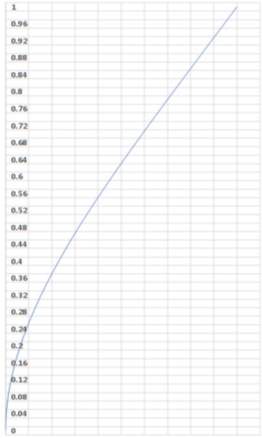
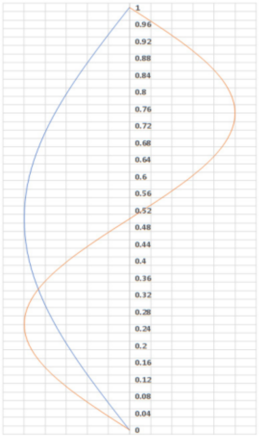
  - A first-order Taylor expansion of the noise source would result in the induced neutron noise (in the frequency-domain) given by the gradient of the Green’s function with respect to the equilibrium position $\mathbf{r}_{p,xy}$ of the moving rod.

Noise source modelling

- Axially-travelling noise source at a velocity v :
  - Noise sources given as "absorber of variable strength" type of noise sources, spatially distributed along the perturbed channel, and time shifted at the location z by $(z - z_0)/v$, with z_0 axial location of the inlet

Noise source modelling

- Fuel assembly vibrations:
  - Different possible axial vibration modes for fuel assemblies:

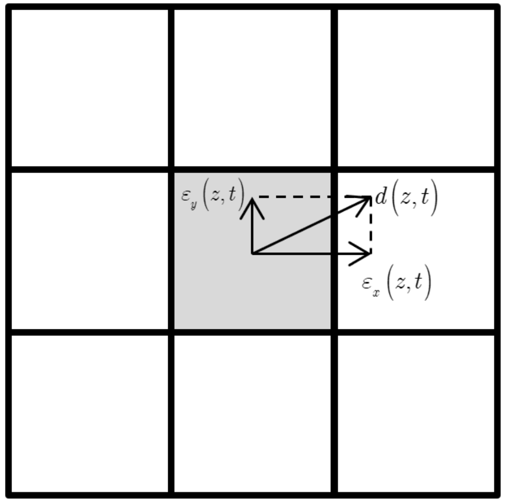
	Cantilevered beam	Simply supported on both sides	Cantilevered beam and simply supported
Axial shape of the displacement $d(z,t)$ in arbitrary units as a function of the relative core elevation z		 first mode in blue, second mode in orange	 first mode in blue, second mode in orange
Oscillation frequency	Ca. 0.6 – 1.2 Hz	Ca. 0.8 – 4 Hz for the first mode Ca. 5 – 10 Hz for the second mode	Ca. 0.8 – 4 Hz for the first mode Ca. 5 – 10 Hz for the second mode

Noise source modelling

- Fuel assembly vibrations:
  - Can be modelled at the pin level: either as “vibrating absorbers” or as “absorbers of variable strength”.
  - Can only be modelled at the nodal level as “absorber of variable strength”.

Noise source modelling

- Fuel assembly vibrations:
  - Lateral vibrations represented as:



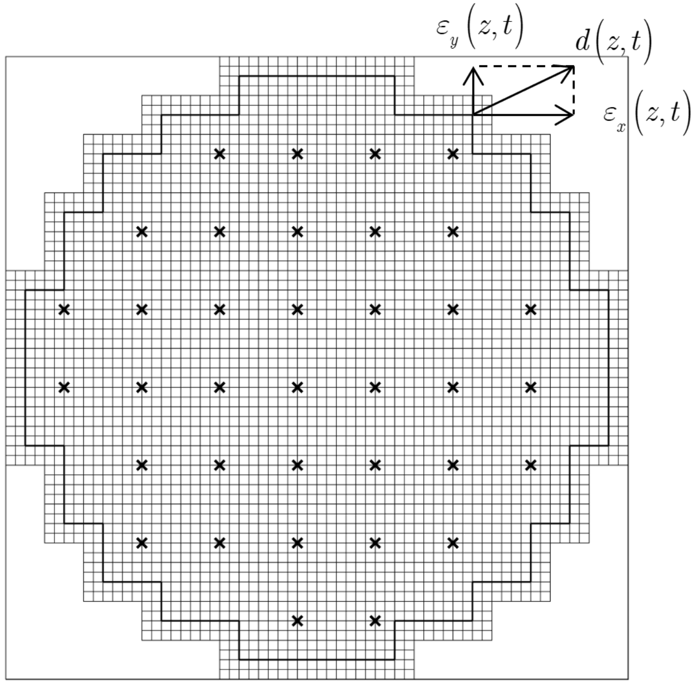
  - The modelling requires a model of the displacements along the x and y directions.

Noise source modelling

- Fuel assembly vibrations:
  - A first-order approximation leads to the modelling of the vibrations along a given direction as two noise sources being out-of-phase and located at the boundary between the vibrating fuel assembly and its two (non-moving) neighbours.
  - A refinement of the mesh to describe the displacement of a structure induces the appearance of higher harmonics.

Noise source modelling

- Core barrel vibrations:
  - Can be seen as a relative displacement of the active core with respect to the reflector:



➢ Modelling identical to the case of vibrating fuel assemblies.

Conclusions and outlook

- Modelling the effect of noise sources can be done in many ways:
  - Time-domain/frequency-domain
  - Diffusion/transport
  - Deterministic/probabilistic
  - Fine/coarse spatial mesh

- Taking full advantage of noise analysis requires:
  - A correct modelling of the noise source
  - The estimation of the reactor transfer function
  - Its inversion

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Backup slides

"Absorber of variable strength" type of noise source

- "Absorber of variable strength" = localized perturbation of which its amplitude varies in time at a fixed position

- Induced neutron noise given by the following balance equation (2-group diffusion theory):

$$\left\{\nabla\cdot\left[\mathbf{D}(\mathbf{r})\nabla\right]+\Sigma_{dyn}(\mathbf{r},\omega)\right\}\times\begin{bmatrix}\delta\phi_1(\mathbf{r},\omega)\\ \delta\phi_2(\mathbf{r},\omega)\end{bmatrix}$$

$$=\boldsymbol{\phi}_r(\mathbf{r})\,\delta\Sigma_r(\mathbf{r},\omega)+\boldsymbol{\phi}_a(\mathbf{r})\begin{bmatrix}\delta\Sigma_{a,1}(\mathbf{r},\omega)\\ \delta\Sigma_{a,2}(\mathbf{r},\omega)\end{bmatrix}+\boldsymbol{\phi}_f(\mathbf{r},\omega)\begin{bmatrix}\delta\nu\Sigma_{f,1}(\mathbf{r},\omega)\\ \delta\nu\Sigma_{f,2}(\mathbf{r},\omega)\end{bmatrix}$$

“Absorber of variable strength” type of noise source

- In case of a point-like source:

$$\left[\nabla_{\mathbf{r}} \cdot \left[\mathbf{D}(\mathbf{r})\nabla_{\mathbf{r}}\right] + \Sigma_{dyn}(\mathbf{r},\omega)\right] \begin{bmatrix} G_{g\rightarrow 1}(\mathbf{r},\mathbf{r}',\omega) \\ G_{g\rightarrow 2}(\mathbf{r},\mathbf{r}',\omega) \end{bmatrix} = \begin{bmatrix} \delta(\mathbf{r}-\mathbf{r}') \\ 0 \end{bmatrix}_{g=1} \text{ or } \begin{bmatrix} 0 \\ \delta(\mathbf{r}-\mathbf{r}') \end{bmatrix}_{g=2}$$

➢ Green’s function

“Absorber of variable strength” type of noise source

- General solution to the original problem can be given by convolution integrals

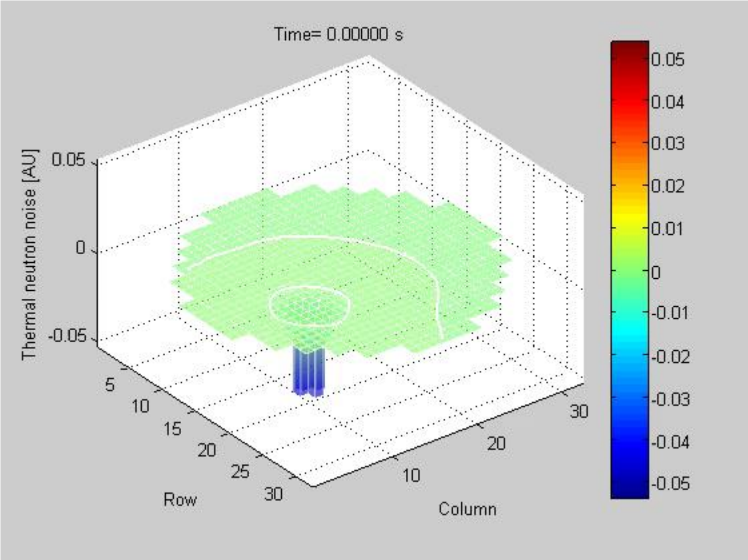
$$\begin{bmatrix} \delta\phi_1(\mathbf{r},\omega) \\ \delta\phi_2(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \int \left[G_{1\to 1}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to 1}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \\ \int \left[G_{1\to 2}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to 2}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \end{bmatrix}$$

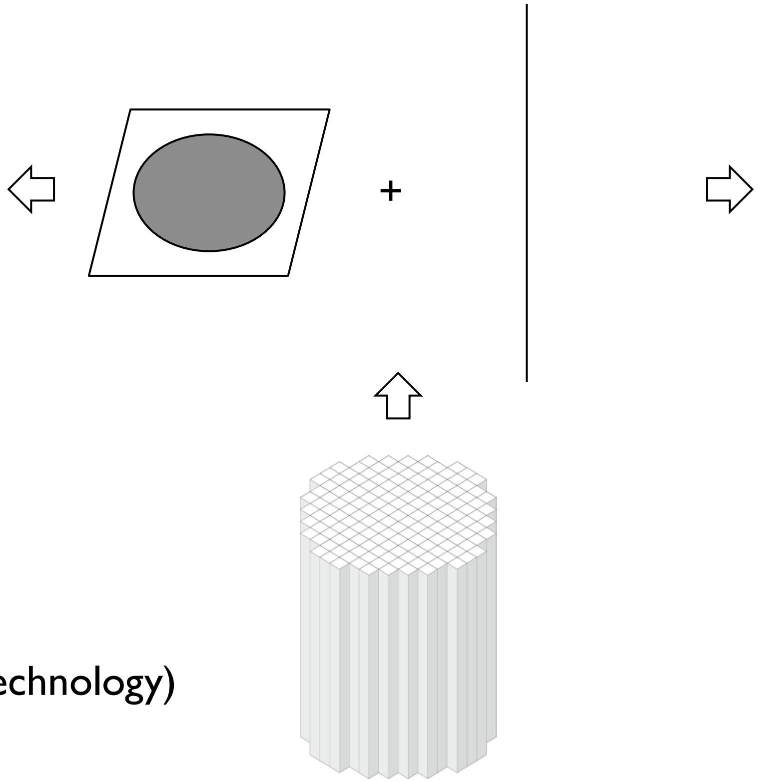
with

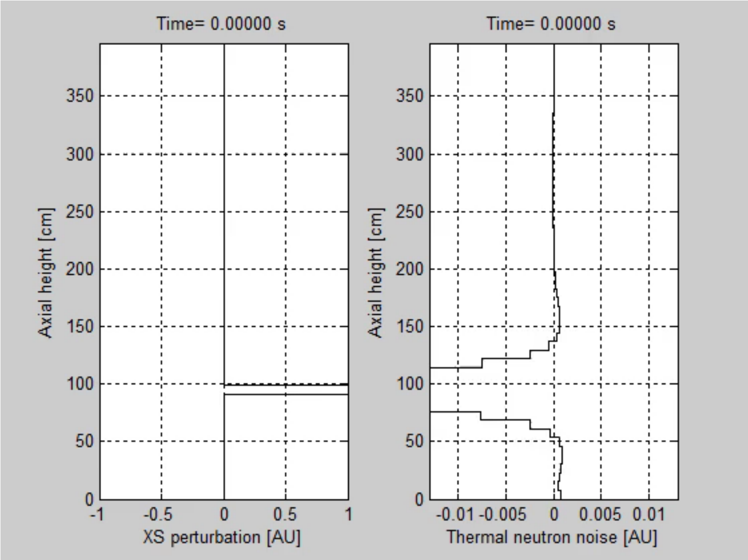
$$\begin{bmatrix} S_1(\mathbf{r}',\omega) \\ S_2(\mathbf{r}',\omega) \end{bmatrix} = \boldsymbol{\phi}_r(\mathbf{r}')\,\delta\Sigma_r(\mathbf{r}',\omega) + \boldsymbol{\phi}_a(\mathbf{r}') \begin{bmatrix} \delta\Sigma_{a,1}(\mathbf{r}',\omega) \\ \delta\Sigma_{a,2}(\mathbf{r}',\omega) \end{bmatrix} + \boldsymbol{\phi}_f(\mathbf{r}',\omega) \begin{bmatrix} \delta\nu\Sigma_{f,1}(\mathbf{r}',\omega) \\ \delta\nu\Sigma_{f,2}(\mathbf{r}',\omega) \end{bmatrix}$$

“Absorber of variable strength” type of noise source

- Example of a localized “absorber of variable strength” @ 1kHz







“Vibrating absorber” type of noise source

- Lateral movement of the absorber represented as (weak absorber):

$$\delta\Sigma_{a,2}(\mathbf{r},t) = \gamma\theta(z - z_0)\left[\delta\left(\mathbf{r}_{xy} - \mathbf{r}_{p,xy} - \boldsymbol{\varepsilon}(t)\right) - \delta\left(\mathbf{r}_{xy} - \mathbf{r}_{p,xy}\right)\right]$$

- A first-order Taylor expansion of the noise source would give for the induced neutron noise (in the frequency-domain):

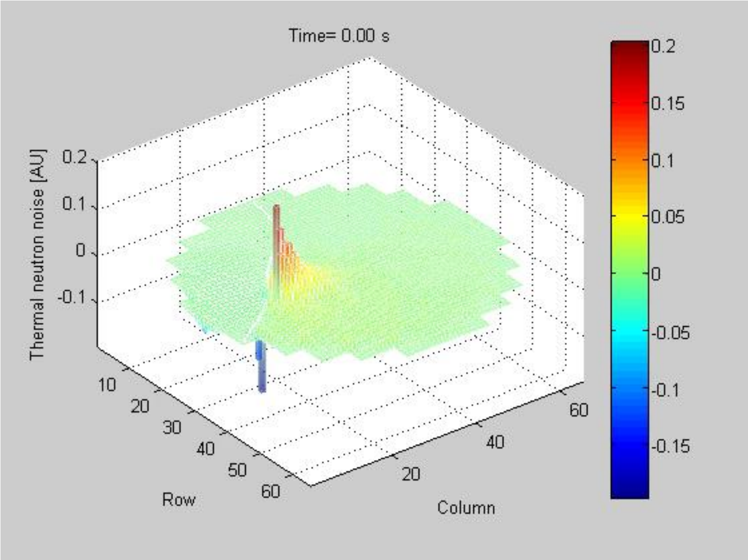
$$\delta\phi_g(\mathbf{r},\omega) = -\gamma\boldsymbol{\varepsilon}(\omega)\cdot\boldsymbol{\delta\psi}_g(\mathbf{r},\omega)$$

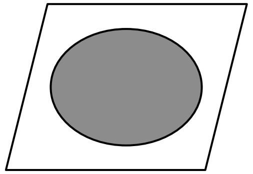
with

$$\boldsymbol{\delta\psi}_g(\mathbf{r},\omega) = \nabla_{\mathbf{r}_{p,xy}}\hat{G}_{2\to g}\left(\mathbf{r},\mathbf{r}_{p,xy},\omega\right)$$

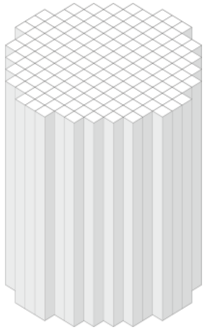
"Vibrating absorber" type of noise source

- Example of a vibrating control rod @ 0.2 Hz





(2-D calculations)



Axially-travelling perturbations

- Noise source represented in the time-domain as:

$$\delta\Sigma_{rem}(\mathbf{r},t) \equiv \delta\Sigma_{rem}(x,y,z,t)$$
$$= \begin{cases} 0, \text{ if } (x,y) \neq (x_0,y_0) \\ 0, \text{ if } (x,y) = (x_0,y_0) \text{ and } z < z_0 \\ \delta\Sigma_{rem}\left(x_0,y_0,z_0,t - \dfrac{z - z_0}{v}\right), \text{ if } (x,y) = (x_0,y_0) \text{ and } z \geq z_0 \end{cases}$$

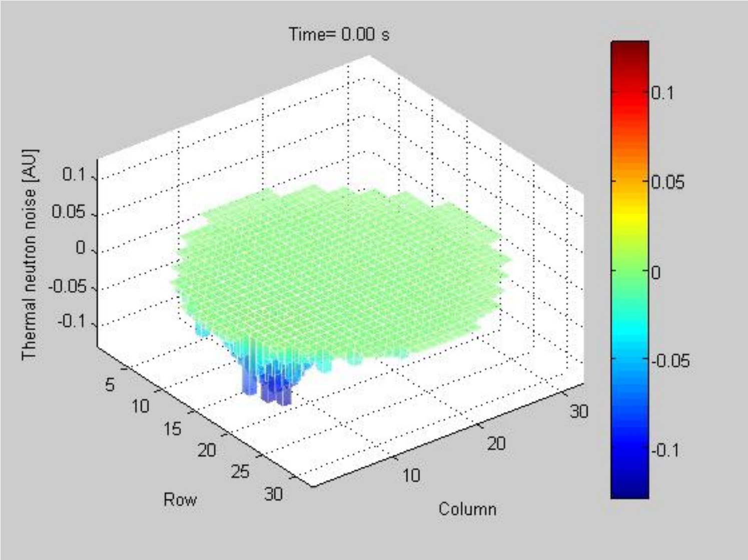
Axially-travelling perturbations

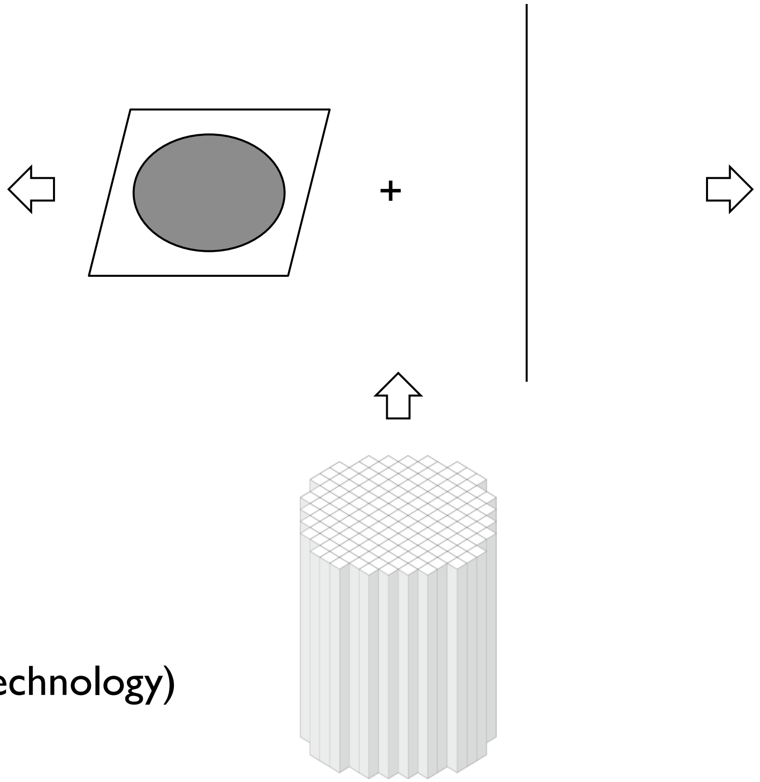
- Noise source represented in the frequency-domain as:

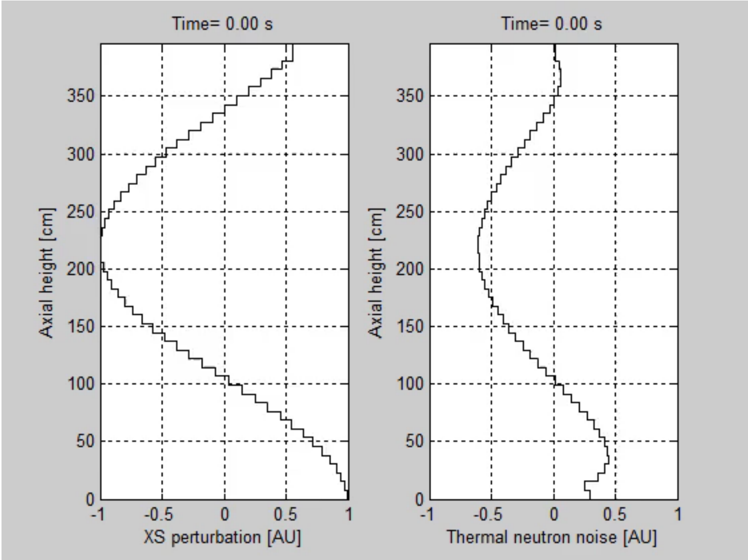
$$\delta\Sigma_{rem}(\mathbf{r},\omega) \equiv \delta\Sigma_{rem}(x,y,z,\omega) = \begin{cases} 0, \text{ if } (x,y) \neq (x_0,y_0) \\ 0, \text{ if } (x,y) = (x_0,y_0) \text{ and } z < z_0 \\ \delta\Sigma_{rem}(x_0,y_0,z_0,\omega)\exp\left[\dfrac{i\omega(z-z_0)}{v}\right] \text{ if } (x,y) = (x_0,y_0) \text{ and } z \geq z_0 \end{cases}$$

Axially-travelling perturbations

- Example of a travelling perturbation @ 1Hz

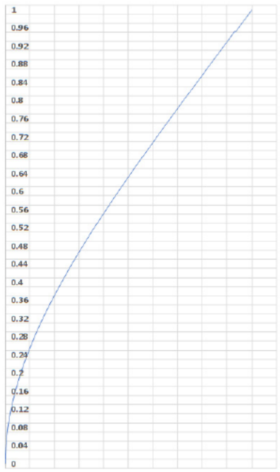
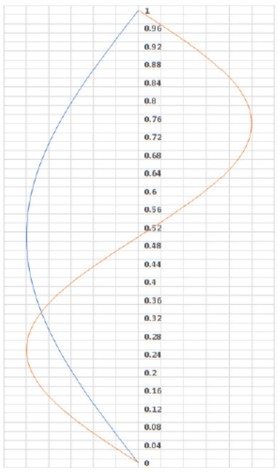
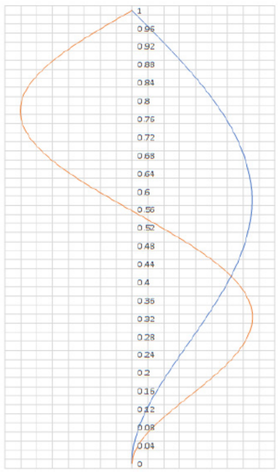






Fuel assembly vibrations

- Different possible axial vibration modes for fuel assemblies:

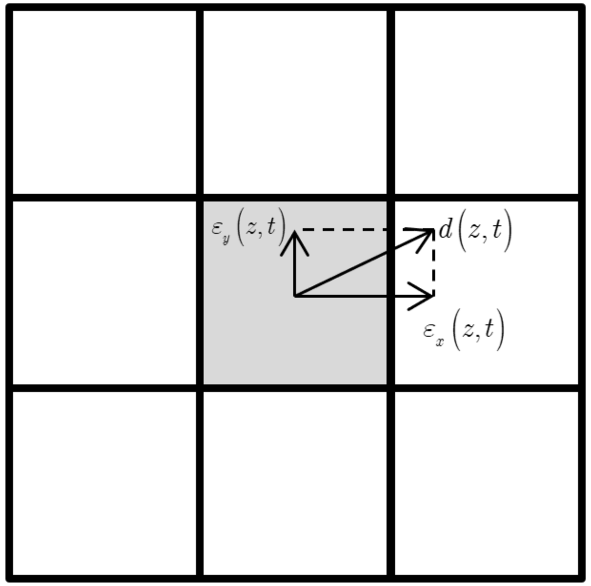
	Cantilevered beam	Simply supported on both sides	Cantilevered beam and simply supported
Axial shape of the displacement $d\left(z,t\right)$ in arbitrary units as a function of the relative core elevation z		 first mode in blue, second mode in orange	 first mode in blue, second mode in orange
Oscillation frequency	Ca. 0.6 – 1.2 Hz	Ca. 0.8 – 4 Hz for the first mode Ca. 5 – 10 Hz for the second mode	Ca. 0.8 – 4 Hz for the first mode Ca. 5 – 10 Hz for the second mode

Fuel assembly vibrations

- Fuel assembly vibrations described at the pin level:
  - Can be modelled as "vibrating absorbers"
  - Can be modelled as "absorbers of variable strength" !
- Fuel assembly vibrations at the nodal level can only be modelled as "absorber of variable strength" !

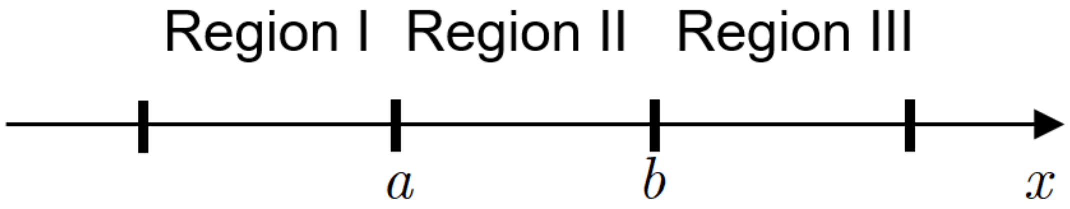
Fuel assembly vibrations

- Lateral vibrations represented as:



Fuel assembly vibrations

- In e.g. the x -direction, one has:



with static cross-section between Regions II and III given as:

$$\Sigma^{x}_{a,g}(x) = \left[1 - \Theta(x - b)\right]\Sigma_{a,g,II} + \Theta(x - b)\Sigma_{a,g,III}$$

Fuel assembly vibrations

- For a time-dependent boundary:

$$b(z,t) = b_0 + \varepsilon_x(z,t)$$

one obtains after a first-order Taylor expansion in the time-domain:

$$\Sigma_{a,g}^{x}(x,z,t) = \left[1 - \Theta(x - b_0)\right]\Sigma_{a,g,II} + \Theta(x - b_0)\Sigma_{a,g,III} + \varepsilon_x(z,t)\delta(x - b_0)\left[\Sigma_{a,g,II} - \Sigma_{a,g,III}\right]$$

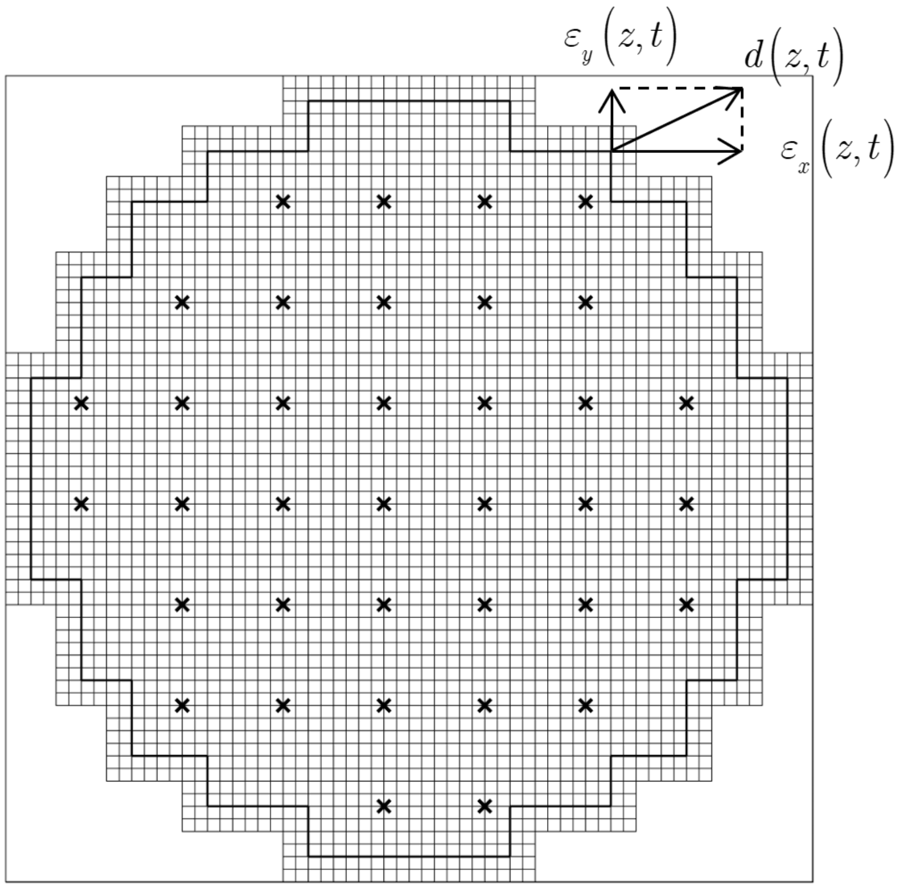
➢ Noise source in the frequency-domain:

$$\delta\Sigma_{a,g}^{x}(x,z,\omega) = \varepsilon_x(z,\omega)\delta(x - b_0)\left[\Sigma_{a,g,II} - \Sigma_{a,g,III}\right]$$

➢ Point-like source!

Pendular core barrel vibrations

- Core barrel vibrations can be seen as a relative displacement of the active core with respect to the reflector:



Pendular core barrel vibrations

- Same technique as for fuel assembly vibrations can be used:

$$\delta\Sigma^{x}_{a,g}(x,z) = h(z)\sum_{n} \delta(x - x_n)\left[\Sigma_{a,g,x_n^-} - \Sigma_{a,g,x_n^+}\right]$$

$$\delta\Sigma^{y}_{a,g}(y,z) = h(z)\sum_{m} \delta(y - y_m)\left[\Sigma_{a,g,y_m^-} - \Sigma_{a,g,y_m^+}\right]$$

- Point-like source!

Estimation of the induced neutron noise

- Generically, the induced neutron noise is given as (e.g. in 2-group theory):

$$\begin{bmatrix} \delta\phi_1(\mathbf{r},\omega) \\ \delta\phi_2(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \int \left[G_{1\to 1}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to 1}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \\ \int \left[G_{1\to 2}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to 2}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \end{bmatrix}$$

with
$$\begin{bmatrix} S_1(\mathbf{r}',\omega) \\ S_2(\mathbf{r}',\omega) \end{bmatrix} = \boldsymbol{\phi}_r(\mathbf{r}')\,\delta\Sigma_r(\mathbf{r}',\omega) + \boldsymbol{\phi}_a(\mathbf{r}') \begin{bmatrix} \delta\Sigma_{a,1}(\mathbf{r}',\omega) \\ \delta\Sigma_{a,2}(\mathbf{r}',\omega) \end{bmatrix} + \boldsymbol{\phi}_f(\mathbf{r}',\omega) \begin{bmatrix} \delta\nu\Sigma_{f,1}(\mathbf{r}',\omega) \\ \delta\nu\Sigma_{f,2}(\mathbf{r}',\omega) \end{bmatrix}$$

Estimation of the induced neutron noise

… or given as:

$$\delta\phi_g(\mathbf{r},\omega) = -g\boldsymbol{\varepsilon}(\omega)\cdot\delta\boldsymbol{\varphi}_g(\mathbf{r},\omega)$$

with

$$\delta\boldsymbol{\varphi}_g(\mathbf{r},\omega) = \nabla_{\mathbf{r}_{p,xy}}\hat{G}_{2\rightarrow g}(\mathbf{r},\mathbf{r}_{p,xy},\omega)$$

- ➢ In essence, only the Green's function is needed

Estimation of the induced neutron noise

- The Green's function can be estimated:
  - Either deterministically
    - Using diffusion theory

$$\left[\nabla_{\mathbf{r}}\cdot\mathbf{D}(\mathbf{r})\nabla_{\mathbf{r}}+\Sigma_{dyn}(\mathbf{r},\omega)\right]\begin{bmatrix}G_{g\to 1}(\mathbf{r},\mathbf{r}',\omega)\\ G_{g\to 2}(\mathbf{r},\mathbf{r}',\omega)\end{bmatrix}=\begin{bmatrix}\delta(\mathbf{r}-\mathbf{r}')\\ 0\end{bmatrix}_{g=1}\text{ or }\begin{bmatrix}0\\ \delta(\mathbf{r}-\mathbf{r}')\end{bmatrix}_{g=2}$$

    - Using transport theory

$$\left[\mathbf{\Omega}\cdot\nabla_{\mathbf{r}}+\Sigma_{dyn}(\mathbf{r},\mathbf{\Omega},\omega)\right]\begin{bmatrix}G_{g\to 1}(\mathbf{r},\mathbf{\Omega},\mathbf{r}',\mathbf{\Omega}',\omega)\\ G_{g\to 2}(\mathbf{r},\mathbf{\Omega},\mathbf{r}',\mathbf{\Omega}',\omega)\end{bmatrix}=\begin{bmatrix}\delta(\mathbf{r}-\mathbf{r}')\delta(\mathbf{\Omega}-\mathbf{\Omega}')\\ 0\end{bmatrix}_{g=1}\text{ or }\begin{bmatrix}0\\ \delta(\mathbf{r}-\mathbf{r}')\delta(\mathbf{\Omega}-\mathbf{\Omega}')\end{bmatrix}_{g=2}$$

Estimation of the induced neutron noise

- The Green's function can also be estimated:
  - Probabilistically:
    - Introducing complex weights in the Monte Carlo solver
    - ➢ Requires modification of the source codes

Estimation of the induced neutron noise

- The Green's function can also be estimated:
  - Probabilistically:
    - Using an equivalence to subcritical problems (demonstrated hereafter on diffusion theory in 2 energy groups):

$$\begin{bmatrix} \delta\phi_1(\mathbf{r},\omega) \\ \delta\phi_2(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \delta\phi_1^{real}(\mathbf{r},\omega) \\ \delta\phi_2^{real}(\mathbf{r},\omega) \end{bmatrix} + i \begin{bmatrix} \delta\phi_1^{im}(\mathbf{r},\omega) \\ \delta\phi_2^{im}(\mathbf{r},\omega) \end{bmatrix}$$

The coupling to the imaginary (real, respectively) part of the neutron noise when solving for the real (imaginary, respectively) balance equations is treated as additional noise sources

$$\begin{bmatrix} S_1^{real\ \text{or}\ im}(\mathbf{r},\omega) \\ S_2^{real\ \text{or}\ im}(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \text{Re or Im}\left\{S_1(\mathbf{r},\omega)\right\} \\ \text{Re or Im}\left\{S_1(\mathbf{r},\omega)\right\} \end{bmatrix} + \mathbf{M} \times \begin{bmatrix} \text{Im or Re}\left\{\delta\phi_1(\mathbf{r},\omega)\right\} \\ \text{Im or Re}\left\{\delta\phi_2(\mathbf{r},\omega)\right\} \end{bmatrix}$$

➢ No modification of the source code required