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## Chapter 2

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### **SMALL SPACE-TIME DEPENDENT FLUCTUA- TIONS: POWER REACTOR NOISE**



## 2.1 Theory of first-order neutron noise

### 2.1.1 General principles

No exercise.

### 2.1.2 Derivation of the first-order neutron noise in one-group diffusion theory

#### Exercise 1:

Derive the balance equation for the neutron noise in linear theory and in the frequency domain assuming that there is no fluctuation in the diffusion coefficient.

### 2.1.3 Derivation of the first-order neutron noise in two-group diffusion theory

#### Exercise 1:

Derive the balance equation for the neutron noise in linear theory and in the frequency domain assuming that there is no fluctuation in the diffusion coefficients.



## 2.2 Theory of first-order neutron noise in its factorized form

### 2.2.1 General principles

No exercise.

### 2.2.2 Determination of the fluctuations of the amplitude factor

#### Exercise 1:

Find the expression of the zero-power reactor transfer function without delayed neutrons. Compare its phase and amplitude to the case with delayed neutrons.

### 2.2.3 Determination of the fluctuations of the shape function

No exercise.



## 2.3 General solution of the neutron noise in one-group diffusion theory

### Exercise 1:

In the case of a homogeneous reactor slab of size  $2a$ , derive the analytical expression of the induced neutron noise for a point-like noise source.

### Exercise 2:

Assuming that the Green's function is known, derive the expression giving the neutron noise induced by a vibrating absorber.

### Exercise 3:

In Cartesian geometry, how to model a travelling perturbation along the  $z$ -axis (localized in  $x$ - $y$  plane) in a core in the time-domain and in the frequency domain? Assume that the perturbation is travelling with coolant flow at a speed  $v$ , that the perturbation has only a significant effect on the (thermal) absorption cross-section and that the perturbation is generated outside of the core.

### Exercise 4:

Considering a one-dimensional system made of two homogeneous regions, how to model a movement of the boundary between those two regions? For that purpose, assume that the perturbation has only a significant effect on the (thermal) absorption cross-section.

### Exercise 5:

In a one-dimensional model of a homogeneous reactor, calculate the Fourier transform of the auto-correlation function of the reactivity effect corresponding to a travelling perturbation along the axis of the reactor. For that purpose, use first order perturbation theory and make use of the Wiener-Khinchin theorem, which states that the Fourier transform of the auto-correlation function of a signal is given by the Fourier transform of the signal times its complex conjugate. Assume that the perturbation is travelling with coolant flow at a speed  $v$ , that the perturbation has only a significant effect on the (thermal) absorption cross-section and that the perturbation is generated outside of the core. The following identity is recalled:

$$\int_{\alpha}^{\beta} \exp(ax) \times \cos(bx) dx = \left[ \exp(ax) \times \frac{a \cos(bx) + b \sin(bx)}{a^2 + b^2} \right]_{\alpha}^{\beta} \quad (2.1)$$

Exercise 6:

Using a one-dimensional homogeneous and infinite system containing a point-like noise source [represented by the fluctuations in the (thermal) macroscopic absorption cross-section], study the space-dependence of the phase shift of the induced neutron noise between two distant spatial points having for abscissa  $x_1$  and  $x_2$  (the origin being taken at the position of the source). Plot as a function of frequency the phase shift in the following case:

$$\nu\Sigma_{f,0} = 0.124 \text{ cm}^{-1} \quad (2.2)$$

$$D_0 = 1.0 \text{ cm}$$

$$\beta = 0.0065$$

$$\lambda = 0.1 \text{ s}^{-1}$$

$$\Lambda_0 = 6.5 \times 10^{-5} \text{ s}^{-1}$$

$$x_1 = 10 \text{ cm}$$

$$x_2 = 40 \text{ cm}$$

## 2.4 General solution of the neutron noise in two-group diffusion theory

### Exercise 1:

In the case of a homogeneous reactor slab of size  $2a$ , derive the analytical expression of the induced neutron noise for a point-like noise source in either the fast group or the thermal group.



## 2.5 Validity of the point-kinetic approximation

### 2.5.1 Case of critical systems

Exercise 1:

Demonstrate that the relative fluctuations in the amplitude factor is also given by the following expression:

$$\frac{\delta P(\omega)}{P_0} = \frac{\int \left[ \frac{1}{v_1} \phi_{1,0}^\dagger(\mathbf{r}) \delta \phi_1(\mathbf{r}, \omega) + \frac{1}{v_2} \phi_{2,0}^\dagger(\mathbf{r}) \delta \phi_2(\mathbf{r}, \omega) \right] d^3 \mathbf{r}}{\int \left[ \frac{1}{v_1} \phi_{1,0}^\dagger(\mathbf{r}) \phi_{1,0}(\mathbf{r}) + \frac{1}{v_2} \phi_{2,0}^\dagger(\mathbf{r}) \phi_{2,0}(\mathbf{r}) \right] d^3 \mathbf{r}} \quad (2.3)$$

### 2.5.2 Case of subcritical systems with an external neutron source

No exercise.



## 2.6 Spatial discretization methods for resolving the neutron noise in nuclear reactors

### Exercise 1:

If one considers the interface between two adjacent one-dimensional homogeneous regions  $n$  and  $n+1$ , as represented in Fig. 2.1, derive the expressions of the neutron current for a finite difference discretization scheme, assuming that the fluxes in the homogeneous problem  $\hat{\phi}_{g,i}^{\mathcal{N}}$  with  $i \in \{n, n+1\}$  are discontinuous as:

$$f_n^+ \hat{\phi}_{g,n}^{\mathcal{N},+} = f_{n+1}^- \hat{\phi}_{g,n+1}^{\mathcal{N},-} \quad (2.4)$$

The neutron current should be expressed as a function of the volume-averaged fluxes in regions  $n$  and  $n+1$ .

In the above equation, the so-called discontinuity factors, defined as:

$$f_n^+ = \frac{\phi_{g,n}^{\mathcal{N},+}}{\hat{\phi}_{g,n}^{\mathcal{N},+}} \quad (2.5)$$

$$f_n^- = \frac{\phi_{g,n}^{\mathcal{N},-}}{\hat{\phi}_{g,n}^{\mathcal{N},-}} \quad (2.6)$$

were introduced, where  $\phi_{g,i}^{\mathcal{N}}$  with  $i \in \{n, n+1\}$  represents the true (non-homogenized) solution, which is continuous at the interface, i.e.

$$\phi_{g,n}^{\mathcal{N},+} = \phi_{g,n+1}^{\mathcal{N},-} \quad (2.7)$$

In the above expressions, the different quantities are defined as:

$$\phi_{g,n}^{\mathcal{N},+} = \phi_{g,n}^{\mathcal{N}} \left( \frac{\Delta \mathcal{N}}{2} \right) \quad (2.8)$$

$$\phi_{g,n+1}^{\mathcal{N},-} = \phi_{g,n+1}^{\mathcal{N}} \left( -\frac{\Delta \mathcal{N}}{2} \right) \quad (2.9)$$

$$\hat{\phi}_{g,n}^{\mathcal{N},+} = \hat{\phi}_{g,n}^{\mathcal{N}} \left( \frac{\Delta \mathcal{N}}{2} \right) \quad (2.10)$$

$$\hat{\phi}_{g,n+1}^{\mathcal{N},-} = \hat{\phi}_{g,n+1}^{\mathcal{N}} \left( -\frac{\Delta \mathcal{N}}{2} \right) \quad (2.11)$$

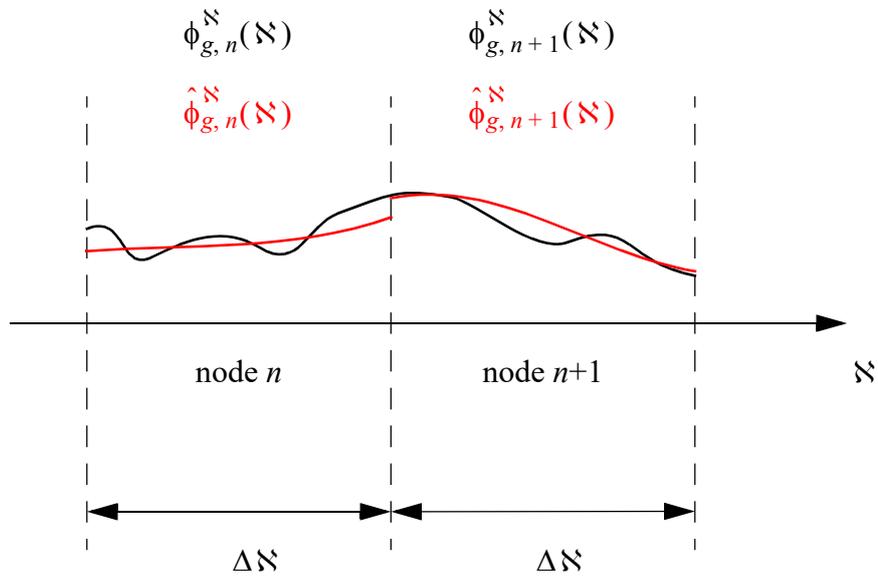


Fig. 2.1 Schematic representation of the true solution  $\phi^{\mathcal{N}}$  in the non-homogenized problem and of the solution  $\hat{\phi}^{\mathcal{N}}$  in the homogenized problem.