Chapter 1

SPACE-TIME DEPENDENT REACTOR KINETICS IN DIFFUSION THEORY

1.1 Static neutron transport

1.1.1 Derivation of the static space-dependent neutron balance equations in diffusion theory

No exercise.

1.1.2 Case of steady-state one-group diffusion theory

Exercise 1

A bare homogeneous spherical thermal reactor has the following characteristics:

$$\begin{aligned} R &= 50 \text{ cm} \\ \upsilon\Sigma_{f,0} &= 0.076 \text{ cm}^{-1} \\ \Sigma_{a,0} &= 0.072 \text{ cm}^{-1} \\ L_0 &= 2.5 \text{ cm} \\ \upsilon &= 2.44 \end{aligned} \quad (1.1)$$

a) Is the reactor critical, subcritical or supercritical?

b) How much increase or decrease of the radius of the reactor is necessary to make the reactor critical?

c) If the reactor radius is kept as $R = 50$ cm, how much increase or decrease of the fissile content is necessary to make the reactor critical, assuming that the ratio between the capture and fission microscopic cross-sections is 0.17 and that the fissile content is restricted to one nuclide only?

Exercise 2

In a three-dimensional Cartesian (x, y, z) coordinate system, the static neutron flux is assumed to be given as the product between three mono-variable functions as:

$$\phi(x, y, z) \equiv X(x)Y(y)Z(z) \quad (1.2)$$

For a reactor having a parallelepiped shape with extrapolated thicknesses a, b, and c in the x-, y-, and z- direction, find the expressions for $X(x)$, $Y(y)$, and $Z(z)$ for a homogeneous reactor in the fundamental mode.

Exercise 3

For the case of one-dimensional homogeneous multiplying system of extrapolated size $2a$ containing a thin absorber rod of strength γ at position x_p, find the expression of the static neutron flux.

1.1.3 Case of steady-state two-group diffusion theory

Exercise 1

Derive the solution to the two-group neutron diffusion equations for a one-dimensional two-region multiplying system. The reactor is made of an active core surrounded by a reflector, as depicted in Fig. 1.1, where a and b correspond to the thickness of the active core and of the reflector, respectively. The origin of the x-axis is chosen in the middle of the system.

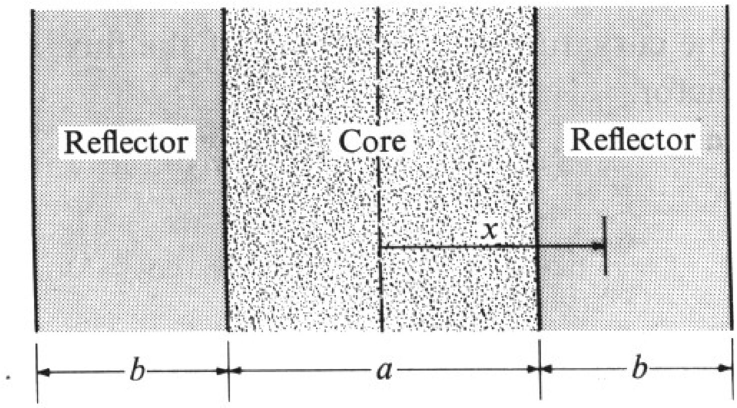

Fig. 1.1 One-dimensional two-region reactor model.

1.2 Dynamic neutron transport

1.2.1 Derivation of the dynamic space-dependent neutron balance equations in diffusion theory

Exercise 1:

Starting from the time-dependent balance equations in multi-group theory and using six groups of delayed neutrons, i.e.

$$\nabla \cdot [D_g(\boldsymbol{r}, t)\nabla\phi_g(\boldsymbol{r}, t)] + \sum_{g'=1}^{G} \Sigma_{s0, g' \to g}(\boldsymbol{r}, t)\phi_{g'}(\boldsymbol{r}, t) \tag{1.3}$$

$$+ (1-\beta)\chi_g^p \sum_{g'=1}^{G} \upsilon\Sigma_{f, g'}(\boldsymbol{r}, t)\phi_{g'}(\boldsymbol{r}, t) + \sum_{i=1}^{6} \lambda_i \chi_{g, i}^d C_i(\boldsymbol{r}, t) - \Sigma_{T, g}(\boldsymbol{r}, t)\phi_g(\boldsymbol{r}, t) = \frac{1}{v_g}\frac{\partial \phi_g}{\partial t}(\boldsymbol{r}, t)$$

$$\frac{\partial C_i}{\partial t}(\boldsymbol{r}, t) = \beta_i \sum_{g'=1}^{G} \upsilon\Sigma_{f, g'}(\boldsymbol{r}, t)\phi_{g'}(\boldsymbol{r}, t) - \lambda_i C_i(\boldsymbol{r}, t),\ i = 1, ..., 6 \tag{1.4}$$

find the expressions these equations reduce to when the time-dependence is neglected.

1.2.2 Case of dynamic one-group diffusion theory

Exercise 1:

For a one-dimensional homogeneous slab of size $2a$ and using one-group of delayed neutrons, find the solution to the inhour equation and thereafter determine the time-dependence of the neutron flux in the following situations:

- A homogeneous perturbation corresponding to a reactivity change of -0.3 \$.
- A homogeneous perturbation corresponding to a reactivity change of +0.3 \$.
- A homogeneous perturbation corresponding to a reactivity change of +1.2 \$.

It is assumed that the core was homogeneous and critical before the perturbation is introduced. The following numerical values are given:

$$\begin{aligned} a &= 150 \text{ cm} \\ \upsilon\Sigma_{f,0} &= 0.0233 \text{ cm}^{-1} \\ \Sigma_{a,0} &= 0.0231 \text{ cm}^{-1} \\ D_0 &= 1.341 \text{ cm} \\ \beta &= 650 \text{ pcm} \\ \lambda &= 0.077 \text{ s}^{-1} \\ t_d &= 210 \times 10^{-6} \text{ s} \end{aligned} \tag{1.5}$$

1.2.3 Case of dynamic two-group diffusion theory

No exercise.

1.3 Resolution of the space- and time-dependence of the neutron flux in nuclear reactors

1.3.1 General discretization methods in space and time in diffusion theory

No exercise.

1.3.2 Reduced Order Modelling (ROM) in diffusion theory

Exercise 1:

Demonstrate that the balance equations in the ROM approach reduce to:

$$\frac{d}{dt}P_m(t) \approx \frac{\rho_m^s - \beta}{\Lambda_m}P_m(t) + \frac{\sum_n \rho_{mn}^f(t)P_n(t)}{\Lambda_m} + \lambda C_m(t) \quad (1.6)$$

and

$$\frac{d}{dt}C_m(t) \approx \frac{\beta}{\Lambda_m}P_m(t) - \lambda C_m(t) \quad (1.7)$$

when neglecting the terms of the form $\beta\left\langle \begin{bmatrix} \phi_{1,m}{}^{\dagger}(\boldsymbol{r}) \\ \phi_{2,m}{}^{\dagger}(\boldsymbol{r}) \end{bmatrix}, \delta\mathbf{F}(\boldsymbol{r},t) \times \begin{bmatrix} \phi_{1,n}(\boldsymbol{r}) \\ \phi_{2,n}(\boldsymbol{r}) \end{bmatrix} \right\rangle$ in comparison with all other terms and when

$$\Lambda_{mn} = \frac{\left\langle \begin{bmatrix} \phi_{1,m}{}^{\dagger}(\boldsymbol{r}) \\ \phi_{2,m}{}^{\dagger}(\boldsymbol{r}) \end{bmatrix}, \mathbf{v}^{-1} \times \begin{bmatrix} \phi_{1,n}(\boldsymbol{r}) \\ \phi_{2,n}(\boldsymbol{r}) \end{bmatrix} \right\rangle}{\left\langle \begin{bmatrix} \phi_{1,m}{}^{\dagger}(\boldsymbol{r}) \\ \phi_{2,m}{}^{\dagger}(\boldsymbol{r}) \end{bmatrix}, \mathbf{F}_0(\boldsymbol{r}) \times \begin{bmatrix} \phi_{1,m}(\boldsymbol{r}) \\ \phi_{2,m}(\boldsymbol{r}) \end{bmatrix} \right\rangle} \ll \Lambda_m, \text{ for } m \neq n \quad (1.8)$$

1.3.3 Flux factorization methods in diffusion theory

Exercise 1:

Demonstrate that the fluctuations of the shape function are orthogonal:

- Either to the static flux in one-group theory:

$$\int \phi_0(\boldsymbol{r})\delta\psi(\boldsymbol{r},t)d^3\boldsymbol{r} = 0 \quad (1.9)$$

- Or to the adjoint functions in two-group theory:

$$\int\left[\frac{1}{v_1}\phi_{1,0}{}^{\dagger}(\boldsymbol{r})\delta\psi_1(\boldsymbol{r},t) + \frac{1}{v_2}\phi_{2,0}{}^{\dagger}(\boldsymbol{r})\delta\psi_2(\boldsymbol{r},t)\right]d^3\boldsymbol{r} = 0 \quad (1.10)$$