CORTEX

Core monitoring techniques and experimental validation and demonstration

Modelling a vibrating absorber in the frequency domain with diffusion and transport theory

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A. Mylonakis (Chalmers), T. Yamamoto (KU), A. Rouchon and A. Zoia (CEA)

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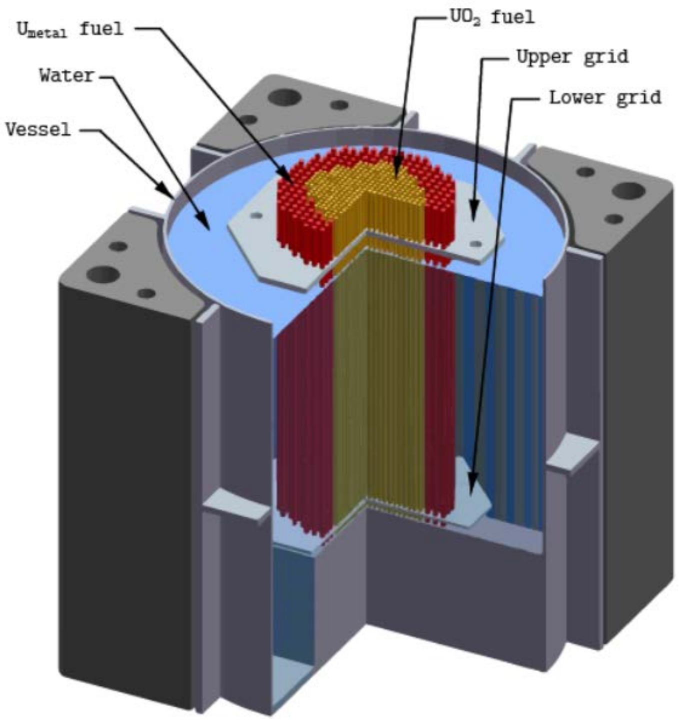
Outline

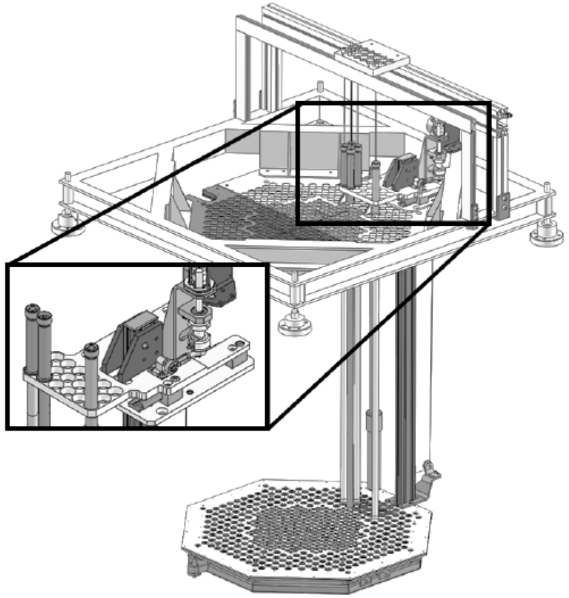
- Introduction: the noise equations in the frequency domain
- The Colibri experiments in CROCUS
- The noise solvers: generalities
- Implementation in CORESIM+, TRIPOLI-4, MCNP

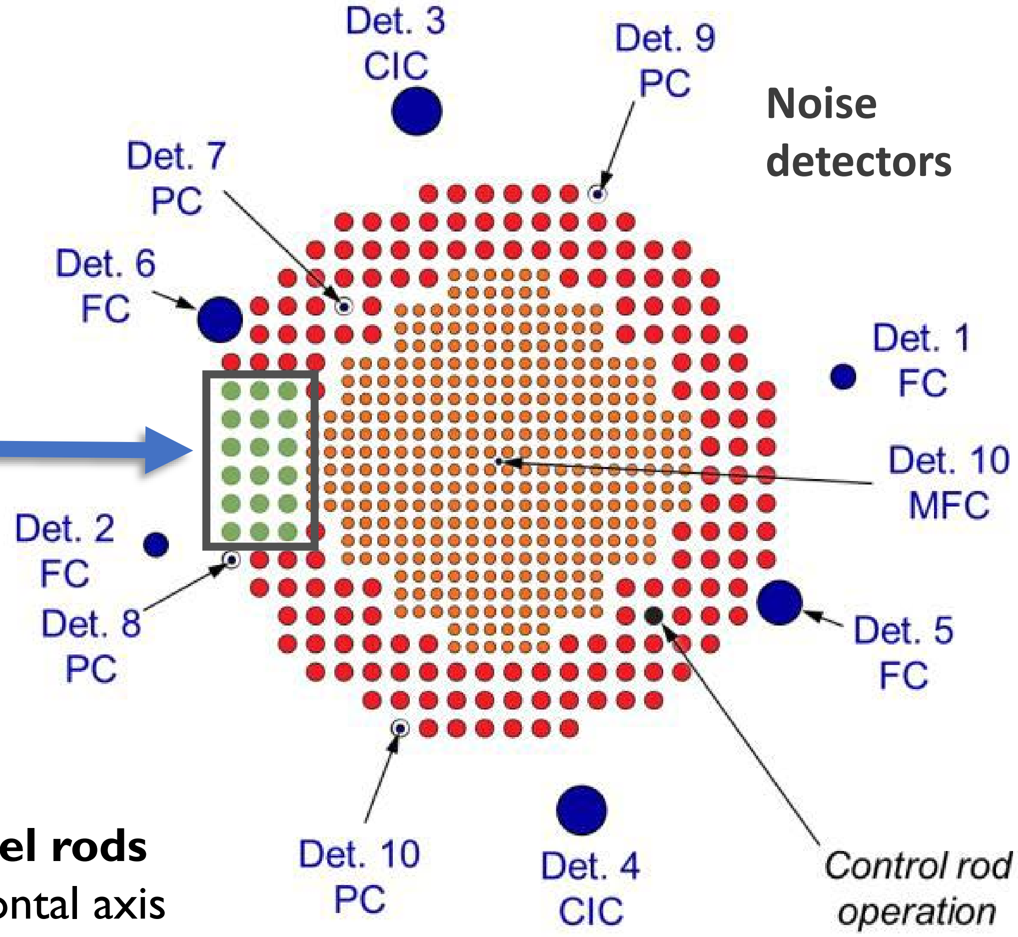
Context: Colibri experiments @CROCUS

The **CROCUS** reactor



The **Colibri** vibrating device



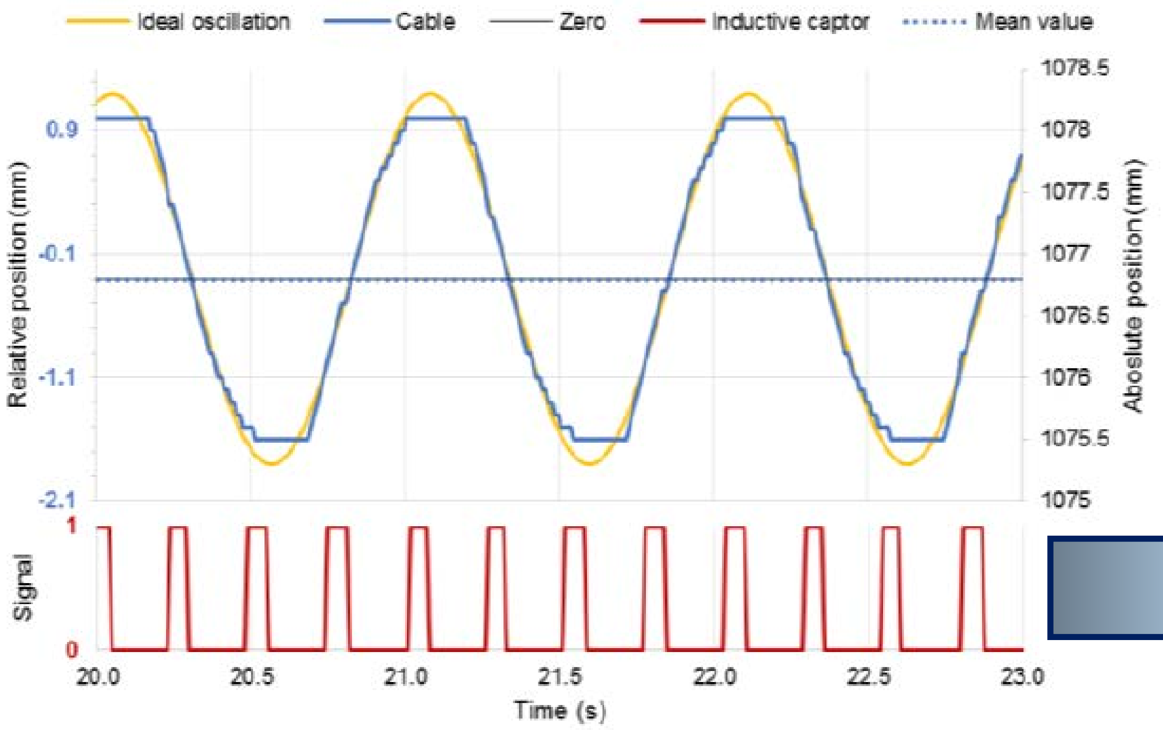


18 vibrating **fuel rods** along the horizontal axis

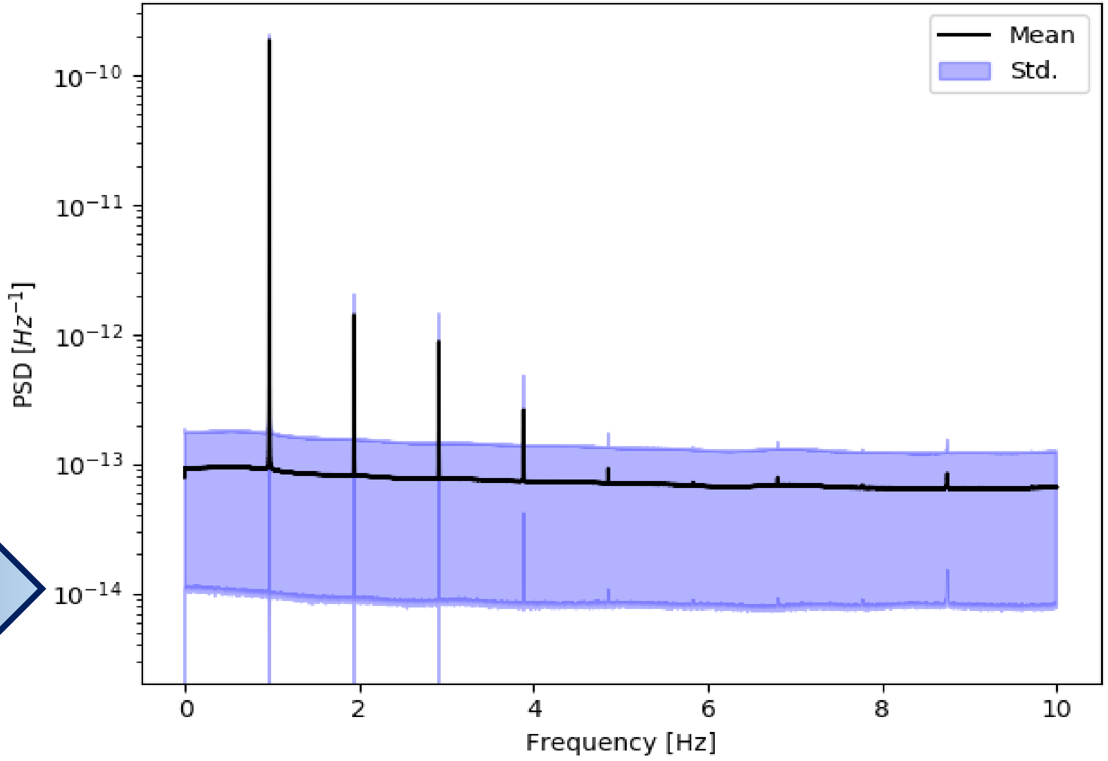
Range of operation : $\omega = [0.1\ -2]\ Hz;\ A = [0.5\ -2]\ mm$

Context: Colibri experiments @CROCUS

Mechanical **vibration** (forcing function)



Measured **noise** (CPSD) in the **frequency domain**



❖ Goal: how to model the **noise** response in the **frequency domain** ?

❖ Specificities of the **vibrations**

The noise equation(s): from time to frequency

Stationary state: $\boxed{\mathcal{B}_c \varphi_c = 0,}$

$$\mathcal{B}_c = \mathbf{\Omega} \cdot \nabla + \Sigma_t - \int \int f_s(\mathbf{\Omega}' \cdot \mathbf{\Omega}, E' \rightarrow E)\Sigma_s(\mathbf{r}, E')dE' d\mathbf{\Omega}'$$
$$- \frac{\chi_p(E)}{4\pi k_{\text{eff}}} \int \int \nu_p(E')\Sigma_f(\mathbf{r}, E')dE' d\mathbf{\Omega}'$$
$$- \sum_j \frac{\chi_d^j(E)}{4\pi k_{\text{eff}}} \int \int \nu_d^j(E')\Sigma_f(\mathbf{r}, E')dE' d\mathbf{\Omega}'$$

Perturbation: $\boxed{\mathcal{B}_p(t)\varphi(t) = 0,}$

$$\mathcal{B}_p(t) = \frac{1}{\upsilon}\frac{\partial}{\partial t} + \mathbf{\Omega} \cdot \nabla + \Sigma_t(\mathbf{r}, E, t)$$
$$- \int \int f_s(\mathbf{\Omega}' \cdot \mathbf{\Omega}, E' \rightarrow E)\Sigma_s(\mathbf{r}, E', t)dE' d\mathbf{\Omega}'$$
$$- \frac{\chi_p(E)}{4\pi k_{\text{eff}}} \int \int \nu_p(E')\Sigma_f(\mathbf{r}, E', t)dE' d\mathbf{\Omega}'$$
$$- \sum_j \lambda_j \frac{\chi_d^j(E)}{4\pi k_{\text{eff}}} \int \int \int e^{-\lambda_j(t-t')} \nu_d^j(E')\Sigma_f(\mathbf{r}, E', t')dE' d\mathbf{\Omega}' dt'$$

Time-dependent cross sections:

$$\Sigma_\alpha(t) = \Sigma_\alpha(\mathbf{r}, E, t)$$

The noise equation(s): from time to frequency

General form of the **perturbed XS**: $\Sigma_\alpha(\mathbf{r}, E, t) = \underbrace{\Sigma_\alpha(\mathbf{r}, E)}_{\text{stationary}} + \underbrace{\delta\Sigma_\alpha(\mathbf{r}, E, t)}_{\text{perturbation}}$

Correspondingly, we have the **flux** decomposition: $\varphi(\mathbf{r}, \mathbf{\Omega}, E, t) = \underbrace{\varphi_c(\mathbf{r}, \mathbf{\Omega}, E)}_{\text{stationary}} + \underbrace{\delta\varphi(\mathbf{r}, \mathbf{\Omega}, E, t)}_{\text{noise}}$

Neutron **noise** equation in the **time domain**: $\boxed{[\mathcal{B}(t) + \delta\mathcal{B}(t)]\delta\varphi(t) = -\delta\mathcal{B}(t)\varphi_c}$ No approximations

$$
\begin{aligned}
\mathcal{B}(t) = {} & \frac{1}{\upsilon}\frac{\partial}{\partial t} + \mathbf{\Omega}\cdot\nabla + \Sigma_t(\mathbf{r}, E) \\
& - \int\int f_s(\mathbf{\Omega}'\cdot\mathbf{\Omega}, E' \rightarrow E)\Sigma_s(\mathbf{r}, E')dE'd\mathbf{\Omega}' \\
& - \frac{\chi_p(E)}{4\pi k_{\text{eff}}}\int\int \nu_p(E')\Sigma_f(\mathbf{r}, E')dE'd\mathbf{\Omega}' \\
& - \sum_j \lambda_j \frac{\chi_d^j(E)}{4\pi k_{\text{eff}}}\int\int\int e^{-\lambda_j(t-t')}\nu_d^j(E')\Sigma_f(\mathbf{r}, E')dE'd\mathbf{\Omega}'dt'
\end{aligned}
$$

$$
\begin{aligned}
\delta\mathcal{B}(t) = {} & \boxed{\delta\Sigma_t(\mathbf{r}, E, t)} - \int\int f_s(\mathbf{\Omega}'\cdot\mathbf{\Omega}, E' \rightarrow E)\boxed{\delta\Sigma_s(\mathbf{r}, E', t)}dE'd\mathbf{\Omega}' \\
& - \frac{\chi_p(E)}{4\pi k_{\text{eff}}}\int\int \nu_p(E')\boxed{\delta\Sigma_f(\mathbf{r}, E', t)}dE'd\mathbf{\Omega}' \\
& - \sum_j \lambda_j \frac{\chi_d^j(E)}{4\pi k_{\text{eff}}}\int\int\int e^{-\lambda_j(t-t')}\nu_d^j(E')\boxed{\delta\Sigma_f(\mathbf{r}, E', t')}dE'd\mathbf{\Omega}'dt'
\end{aligned}
$$

The noise equation(s): from time to frequency

Fourier transform: $f(\omega) = \mathcal{F}[f(t)](\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} f(t)dt$

$\mathcal{B}(t)$ ⇒

$$\mathcal{B}(\omega) = i\frac{\omega}{v} + \Sigma_t(\mathbf{r},E) + \mathbf{\Omega}\cdot\nabla - \int\int f_s(\mathbf{\Omega}'\cdot\mathbf{\Omega},E'\to E)\Sigma_s(\mathbf{r},E')dE'd\mathbf{\Omega}'$$
$$-\frac{\chi_p(E)}{4\pi k_{\mathrm{eff}}}\int\int \nu_p(E')\Sigma_f(\mathbf{r},E')dE'd\mathbf{\Omega}'$$
$$-\sum_j \frac{\lambda_j}{\lambda_j+i\omega}\frac{\chi_d^j(E)}{4\pi k_{\mathrm{eff}}}\int\int \nu_d^j(E')\Sigma_f(\mathbf{r},E')dE'd\mathbf{\Omega}'$$

Frequency domain operators

$\delta\mathcal{B}(t)$ ⇒

$$\delta\mathcal{B}(\omega) = \delta\Sigma_t(\mathbf{r},E,\omega) - \int\int f_s(\mathbf{\Omega}'\cdot\mathbf{\Omega},E'\to E)\delta\Sigma_s(\mathbf{r},E',\omega)dE'd\mathbf{\Omega}'$$
$$-\frac{\chi_p(E)}{4\pi k_{\mathrm{eff}}}\int\int \nu_p(E')\delta\Sigma_f(\mathbf{r},E',\omega)dE'd\mathbf{\Omega}'$$
$$-\sum_j \frac{\lambda_j}{\lambda_j+i\omega}\times\frac{\chi_d^j(E)}{4\pi k_{\mathrm{eff}}}\int\int \nu_d^j(E')\delta\Sigma_f(\mathbf{r},E',\omega)dE'd\mathbf{\Omega}'.$$

Orthodox linearization in the Fourier domain

Exact formulation in the Fourier domain :

$$[\mathcal{B}(t) + \delta\mathcal{B}(t)]\delta\varphi(t) = -\delta\mathcal{B}(t)\varphi_c \quad \Rightarrow \quad \mathcal{B}(\omega)\delta\varphi(\omega) + \frac{1}{2\pi}\int \delta\mathcal{B}(\omega-\omega')\delta\varphi(\omega')d\omega' = -\delta\mathcal{B}(\omega)\varphi_c$$

***Orthodox* linearization** (neglect the product of two perturbed terms):

$$\Rightarrow \quad \underbrace{\mathcal{B}(\omega)}_{\text{«Transport operator»}}\delta\varphi(\omega) = \underbrace{-\delta\mathcal{B}(\omega)\varphi_c}_{\text{«Noise source»}}$$

The **linearized noise equation** has the structure of a *fixed-source transport equation* for the unknown noise field φ(ω)

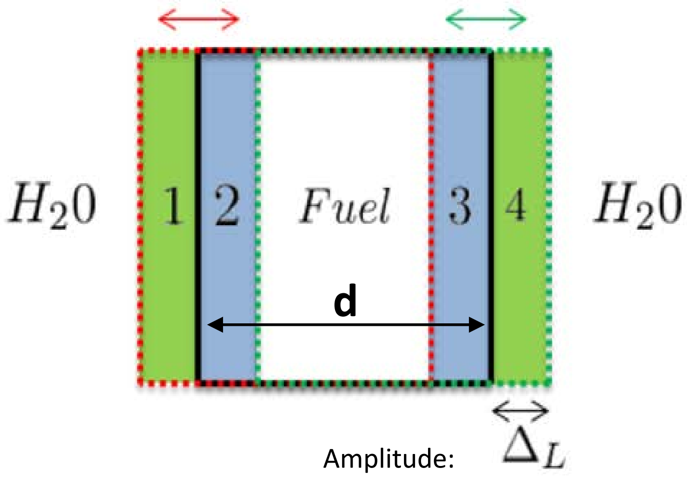
- The **source** depends on the critical flux φ_c
- **B(ω)** is a complex Boltzmann-like operator
- **δB(ω)** depends on the kind of perturbation

Modeling the noise source in the frequency domain

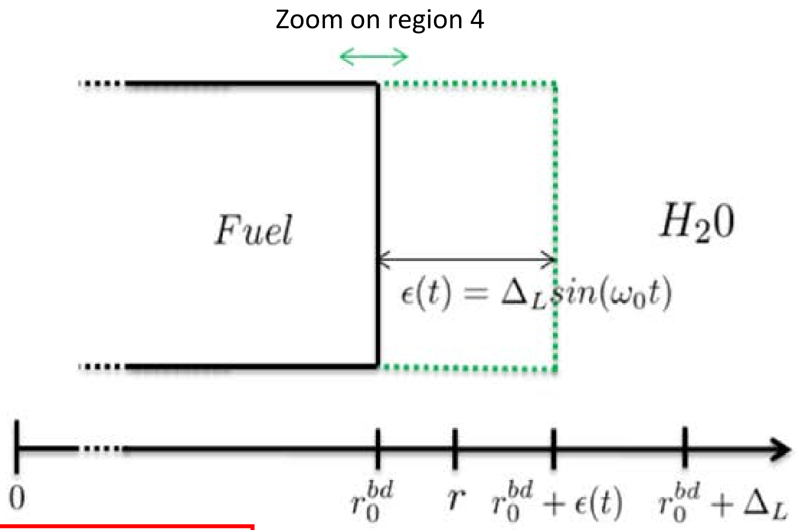
The **noise source** $-\delta\mathcal{B}(\omega)\varphi_c$ contains **terms** of the kind: $\delta\Sigma_\alpha(\mathbf{r}, E, \omega)$

Let us examine the case of **mechanical vibrations**:

Vibration of a **fuel pin** of **size d >** Δ_L



Amplitude: Δ_L



Time behaviour of the perturbed cross sections: **a moving interface**

For region 4: $\frac{\delta\Sigma_4(t)}{\Sigma_4} = \frac{\Sigma_{\text{fuel}} - \Sigma_{\text{H}_2\text{O}}}{\Sigma_{\text{H}_2\text{O}}} H(r_0^{bd} + \varepsilon(t) - r)$

Heaviside step function

The noise source due to a moving interface

$$\delta\Sigma_r(x,E,t) = \Delta\Sigma_r(E)\left[H(x-x_0) - H(x-x_0-\varepsilon\sin(\omega_0 t))\right]$$

Exact Fourier transform

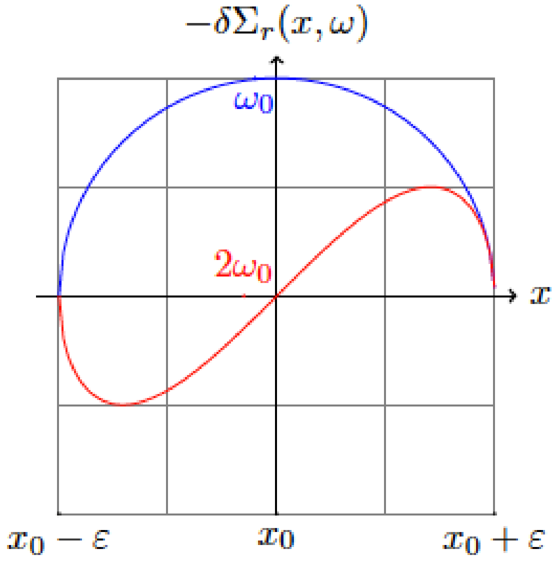
$$\delta\Sigma_r^R(x,E,\omega) = \Delta\Sigma_r(E)\left\{c_0^R(x,x_0)\delta(\omega) + \sum_{k=1}^{\infty} c_k^R(x,x_0)\left[\delta(\omega-k\omega_0)+\delta(\omega+k\omega_0)e^{ik\pi}\right]\right\}$$

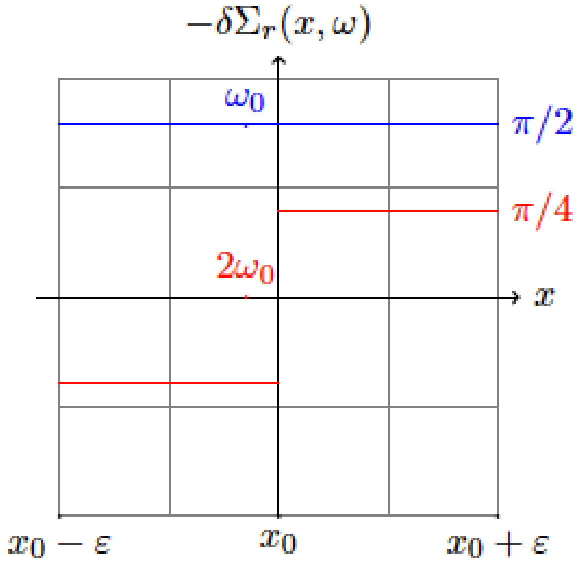
ε/d approximation

$$c_n^R(x,x_0) = 2\frac{\sin\left(n\arccos\left(\frac{x-x_0}{\varepsilon}\right)\right)}{n}e^{-in\frac{\pi}{2}}$$

$$\delta\Sigma_r(x,E,\omega) = \Delta\Sigma_r(E)\left[-i\pi\varepsilon\delta(x-x_0)\delta(\omega-\omega_0) + \frac{\pi}{4}\varepsilon^2\delta'(x-x_0)\delta(\omega-2\omega_0)+\cdots\right]$$

- The source is **complex** and contains an **infinite number of harmonics**
- «Typically»: keep only the **first harmonic** at $\omega = \omega_0$





Calculation strategy

1. Determine the **critical flux φ_c**
2. Prepare **$-\delta B(\omega)$** for any ω, depending on the specific model and assumptions
3. Determine the **noise source $-\delta B(\omega)\,\varphi_c$**
4. Solve the **noise equation** $B(\omega)\,\delta\varphi(\omega) = -\delta B(\omega)\,\varphi_c$ and determine the **noise field $\delta\varphi(\omega)$**
5. Compute **derived quantities**, e.g. CPSDs, etc., based on the noise field $\delta\varphi(\omega)$

- Specific implementations (and possibly approximations) are code-dependent:
  - Monte Carlo vs. Deterministic
  - Transport vs. Diffusion
  - Exact source vs. ε/d approximation
- In the following: CORE SIM+, TRIPOLI-4, MCNP

CORE SIM+

Noise equations CORE SIM+ solves

$$\left[\nabla \cdot \boldsymbol{D}(\boldsymbol{r})\nabla + \boldsymbol{\Sigma}_{dyn}^{crit}(\boldsymbol{r},\omega)\right] \times \underbrace{\begin{bmatrix}\delta\phi_1(\boldsymbol{r},\omega)\\ \delta\phi_2(\boldsymbol{r},\omega)\end{bmatrix}}_{\text{Neutron noise}} =$$

$$= \underbrace{\boldsymbol{\phi}_r\,(\boldsymbol{r})\delta\Sigma_r(\boldsymbol{r},\omega) + \boldsymbol{\phi}_a(\boldsymbol{r})\begin{bmatrix}\delta\Sigma_{a,1}(\boldsymbol{r},\omega)\\ \delta\Sigma_{a,2}(\boldsymbol{r},\omega)\end{bmatrix} + \boldsymbol{\phi}_f^{crit}(\boldsymbol{r},\omega)\begin{bmatrix}\delta\upsilon\Sigma_{f,1}(\boldsymbol{r},\omega)\\ \delta\upsilon\Sigma_{f,2}(\boldsymbol{r},\omega)\end{bmatrix}}_{\text{Noise sources}}$$

Approximations:
1. Diffusion theory & 2 energy groups
2. Small perturbations
3. Products of small terms are neglected

Finite volume formulation & finite difference spatial discretization → system of algebraic equations:

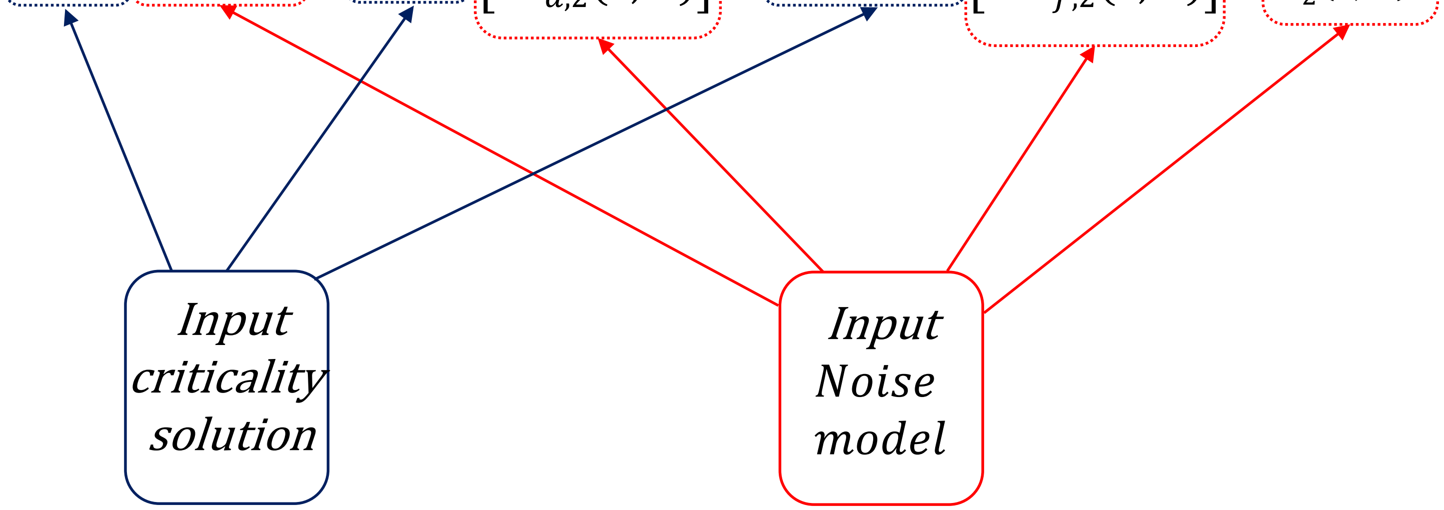
$$\boldsymbol{A}_{noise}\boldsymbol{\Phi}_{noise} = \boldsymbol{S}_{noise}$$

Noise source modelling

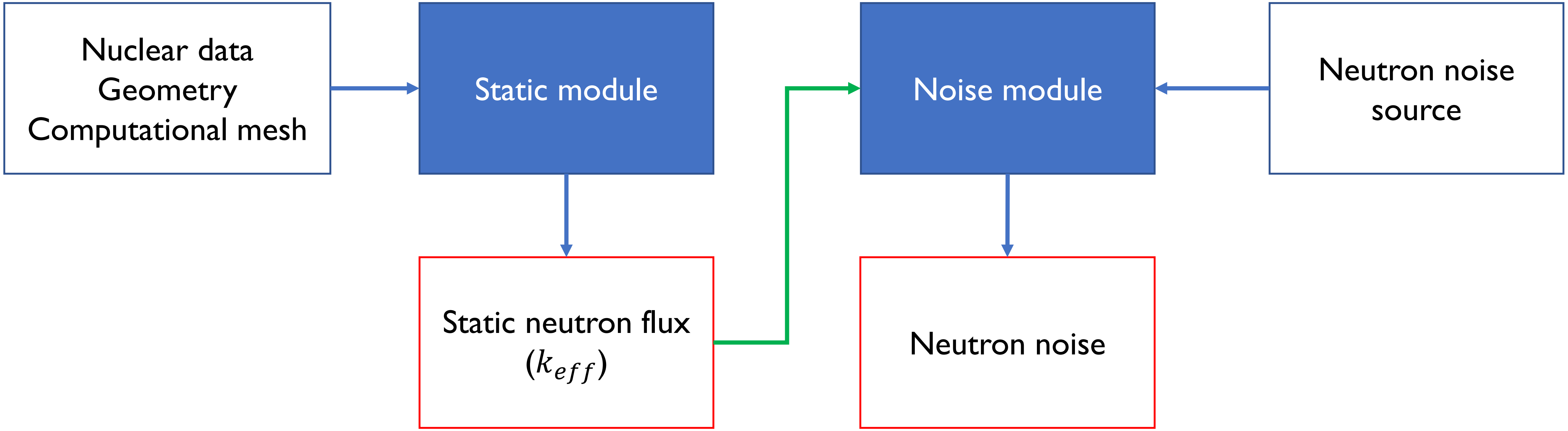
$$\boldsymbol{A}_{noise}\boldsymbol{\Phi}_{noise} = \boldsymbol{S}_{noise}$$

The noise source models are built based on the XS fluctuations or arbitrary source terms:

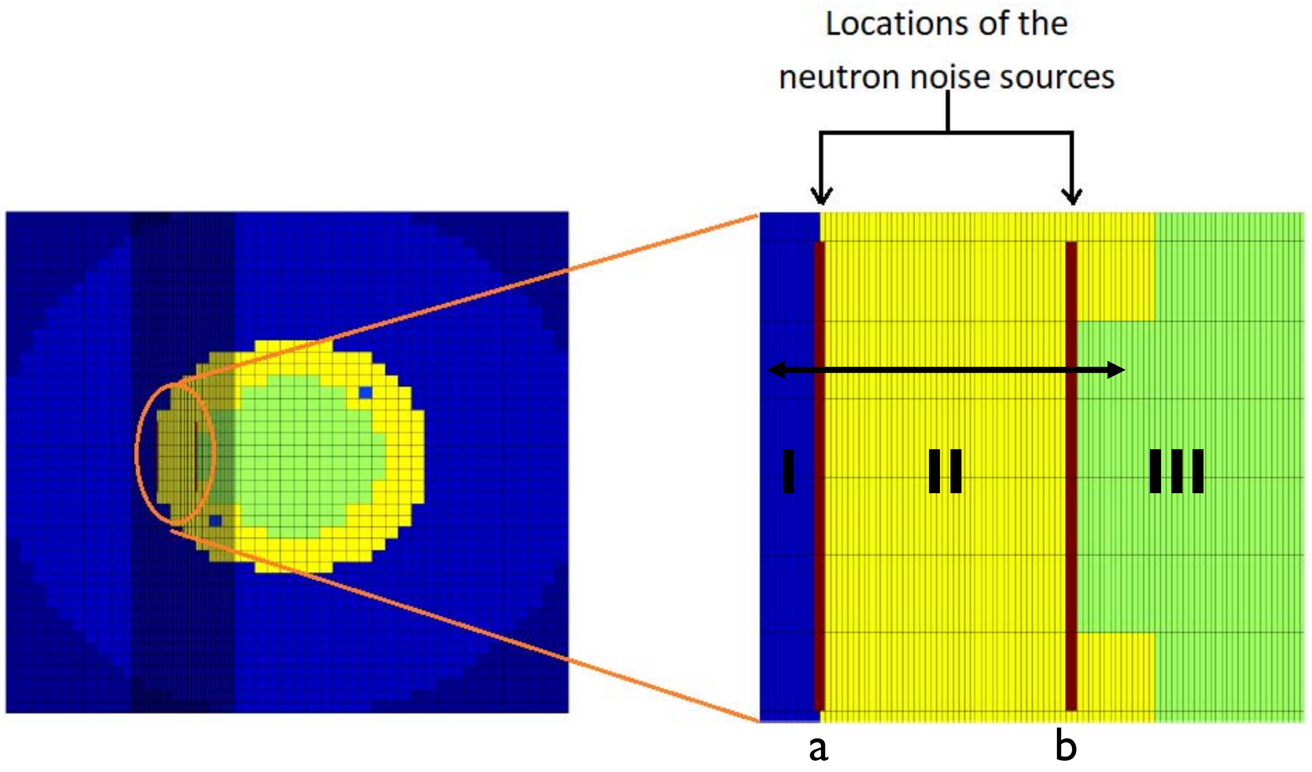
$$\boldsymbol{S}_{noise} = \begin{bmatrix} S_1(\boldsymbol{r},\omega) \\ S_2(\boldsymbol{r},\omega) \end{bmatrix} = \boldsymbol{\phi}_r(\boldsymbol{r})\,\delta\Sigma_r(\boldsymbol{r},\omega) + \boldsymbol{\phi}_a(\boldsymbol{r}) \begin{bmatrix} \delta\Sigma_{a,1}(\boldsymbol{r},\omega) \\ \delta\Sigma_{a,2}(\boldsymbol{r},\omega) \end{bmatrix} + \boldsymbol{\phi}_f^{crit}(\boldsymbol{r},\omega) \begin{bmatrix} \delta\upsilon\Sigma_{f,1}(\boldsymbol{r},\omega) \\ \delta\upsilon\Sigma_{f,2}(\boldsymbol{r},\omega) \end{bmatrix} + \begin{bmatrix} s_1(\boldsymbol{r},\omega) \\ s_2(\boldsymbol{r},\omega) \end{bmatrix}$$



General scheme of CORE SIM+



CROCUS and COLIBRI model



ε/d model

$$\delta\Sigma^{x}_{\alpha,g}(r,\omega) = \epsilon(\omega)\delta(r-r_{\mathrm{a}})\left[\Sigma_{\alpha,g,I}-\Sigma_{\alpha,g,II}\right] + \epsilon(\omega)\delta(r-r_{\mathrm{b}})\left[\Sigma_{\alpha,g,II}-\Sigma_{\alpha,g,III}\right]$$

Input Noise model

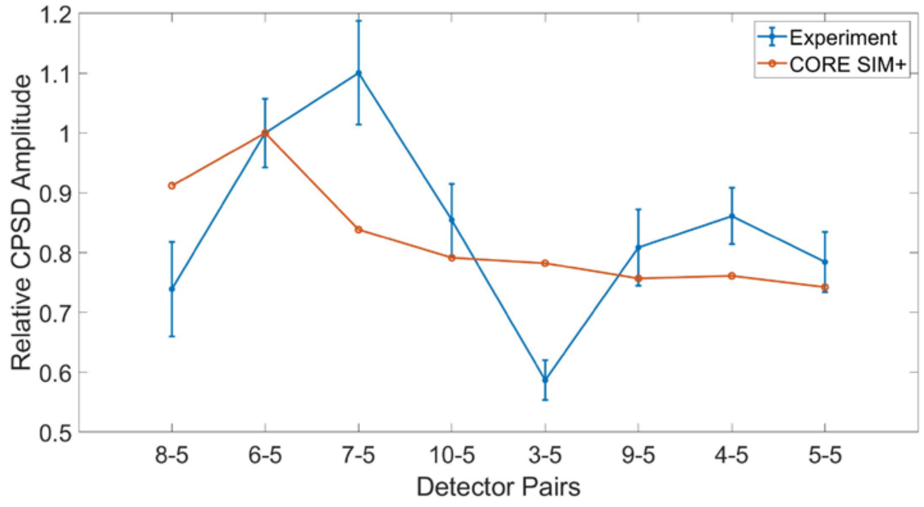
Computational performance for COLIBRI simulations

Tight convergence criteria

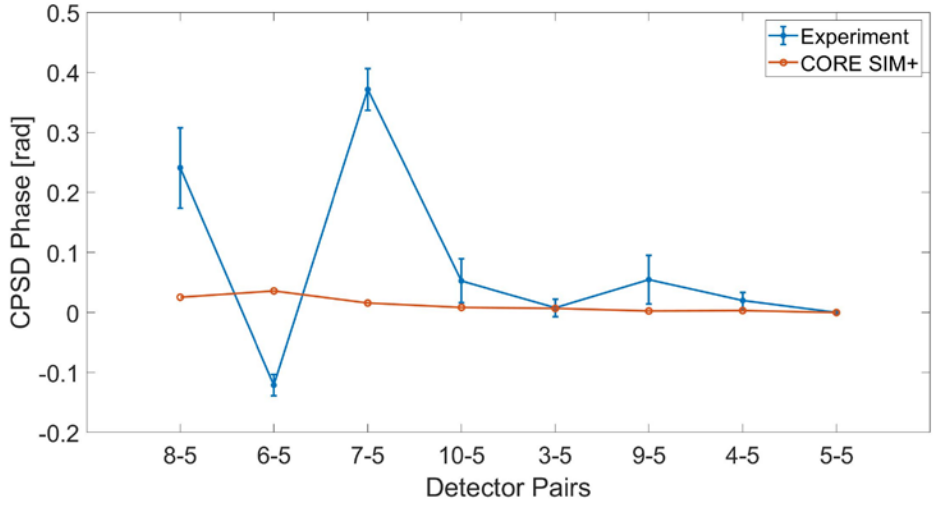
Solver	*Mesh*	*Method*	*Time*
Eigenvalue	Non-uniform	PI-Cheb-GMRES-ILU(0)	5.5 min
Noise	Non-uniform	GMRES - ILU(0)	26 s

Experiment 12 – CORE SIM+

Amplitude

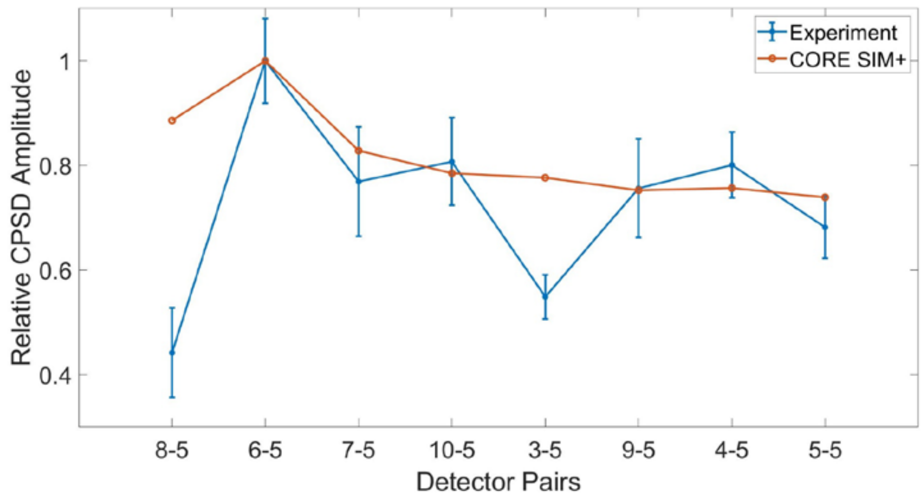


Phase

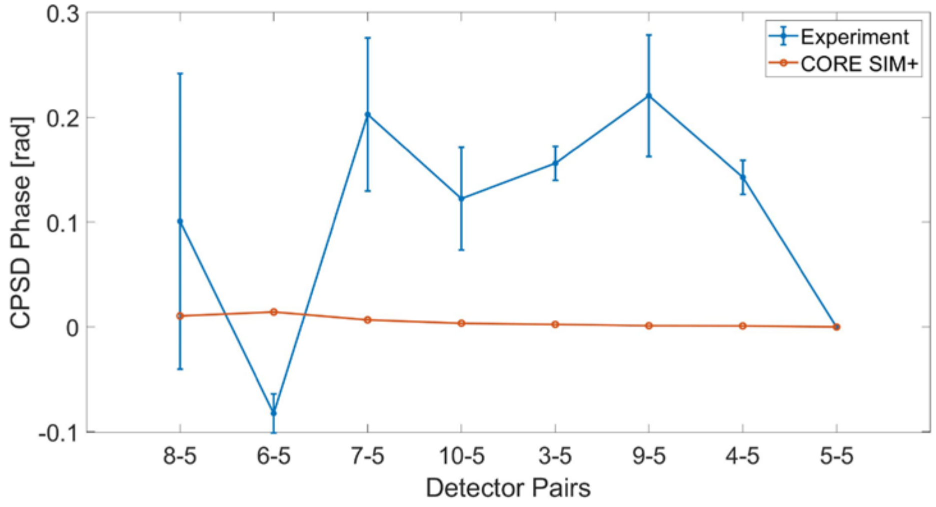


Experiment 13 – CORE SIM+

Amplitude



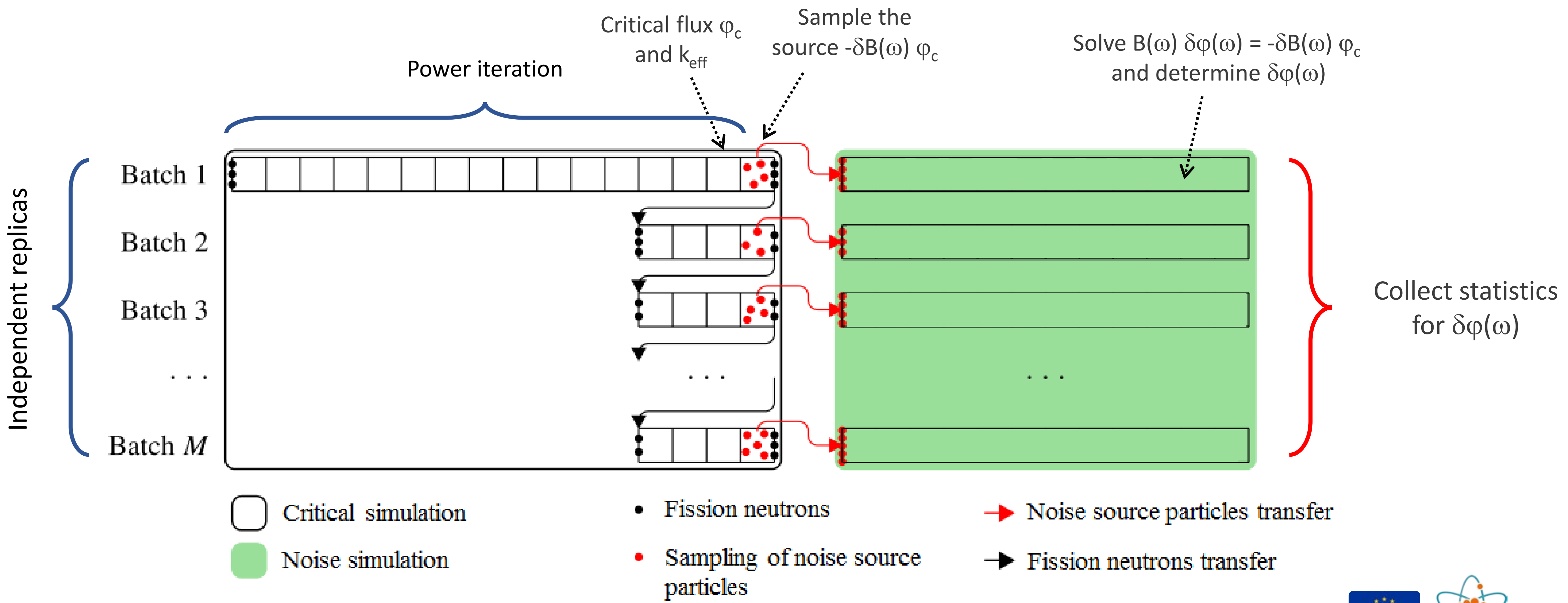
Phase



TRIPOLI-4

Scheme of the noise calculation

Compute the critical flux → sample the noise source → compute the noise field



Complex particles

Emission spectrum — **Complex** weight correction — Reaction rate

The **noise source** $-\delta B(\omega)\,\varphi_c$ contains terms of the kind:

$$f_r(\mathbf{\Omega}, E \to \mathbf{\Omega}', E') \times \frac{\delta\Sigma_r(\mathbf{r}, E, \omega)}{\Sigma_r(\mathbf{r}, E)} \times \Sigma_r(\mathbf{r}, E)\,\varphi_c(\mathbf{r}, \mathbf{\Omega}, E)$$

"Noise particles" are assumed to have **complex weights** (with sign): $w = w_R + i w_I$

Follow 1 noise particle with 2 weights: real (w_R) and imaginary (w_I)

$$c_n^R(x, x_0) = 2\frac{\sin\left(n \arccos\left(\frac{x - x_0}{\varepsilon}\right)\right)}{n} e^{-in\frac{\pi}{2}}$$

Once the source is known, we have to **solve** $B(\omega)\,\delta\varphi(\omega) = -\delta B(\omega)\,\varphi_c$

Particle transport is ruled by the complex Boltzmann-like operator $B(\omega)$

$$\mathcal{B}(\omega) = i\frac{\omega}{v} + \Sigma_t(\mathbf{r}, E) + \mathbf{\Omega}\cdot\nabla - \int\int f_s(\mathbf{\Omega}'\cdot\mathbf{\Omega}, E' \to E)\Sigma_s(\mathbf{r}, E')dE'd\mathbf{\Omega}'$$
$$- \frac{\chi_p(E)}{4\pi k_{\text{eff}}}\int\int \nu_p(E')\Sigma_f(\mathbf{r}, E')dE'd\mathbf{\Omega}'$$
$$- \sum_j \frac{\lambda_j}{\lambda_j + i\omega}\frac{\chi_d^j(E)}{4\pi k_{\text{eff}}}\int\int \nu_d^j(E')\Sigma_f(\mathbf{r}, E')dE'd\mathbf{\Omega}'$$

Build a **random walk** based on flights and collisions:

- Leakage, absorption and scattering: usual Monte Carlo rules apply (to both real and imaginary particles)
- Imaginary absorption and fission: special treatment

Modified collision events

- The modified fission term: $F_\omega = F_p + \sum_j \frac{\lambda_j}{\lambda_j + i\omega} F_d^j$

Delayed fission

Prompt fission

Complex **yield multiplier** depending on the precursor decay constant of family j

- The imaginary absorption term:

$$\frac{i\omega}{v} + \Sigma_{t,0} = \cdots \quad \Rightarrow \quad \Sigma_{t,0} + \frac{\omega}{v} = \int\int \nu_\omega \frac{\omega}{v'} \delta(\mathbf{\Omega} - \mathbf{\Omega}')\delta(E - E')d\mathbf{\Omega}'dE'$$

Modified total cross section

Complex yield: $\nu_\omega = 1 - i$

Real absorption cross section

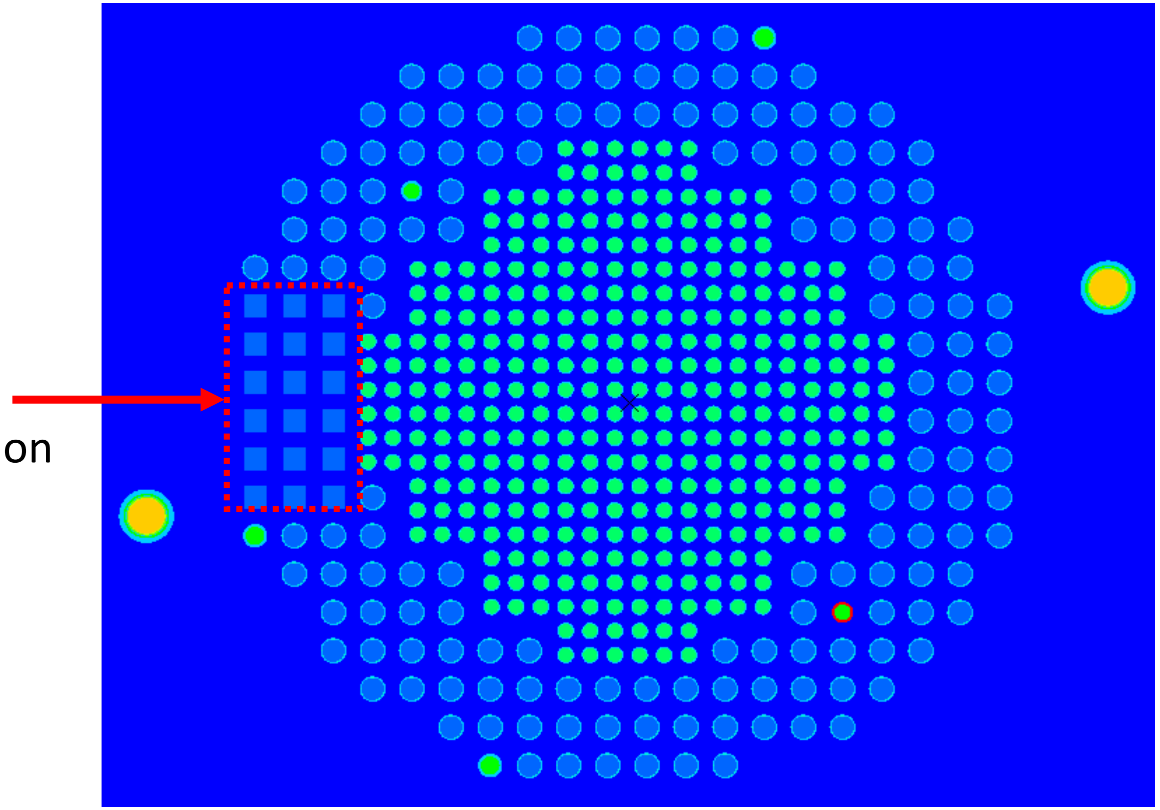
Associated (copy) production term

- Standard Monte Carlo methods can be used
  - Implicit capture, forced fission, population control (Russian roulette & splitting)

    Each applied separately on the real and imaginary weight

The Crocus model for Tripoli-4

Square fuel pins for the vibrating region



Continuous-energy treatment
JEFF3.1.1 nuclear data

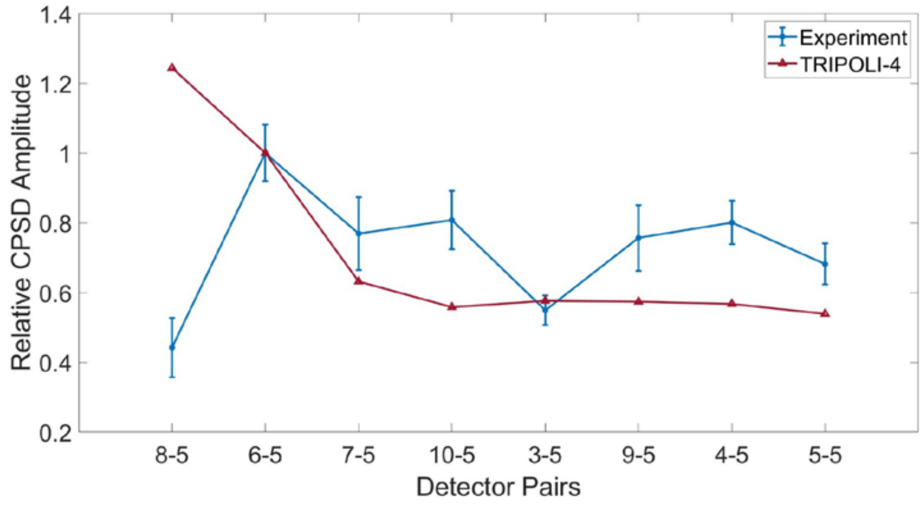
Fully detailed 3D model
Detectors explicitly described

Noise field computed over a spatial mesh and in the detector regions

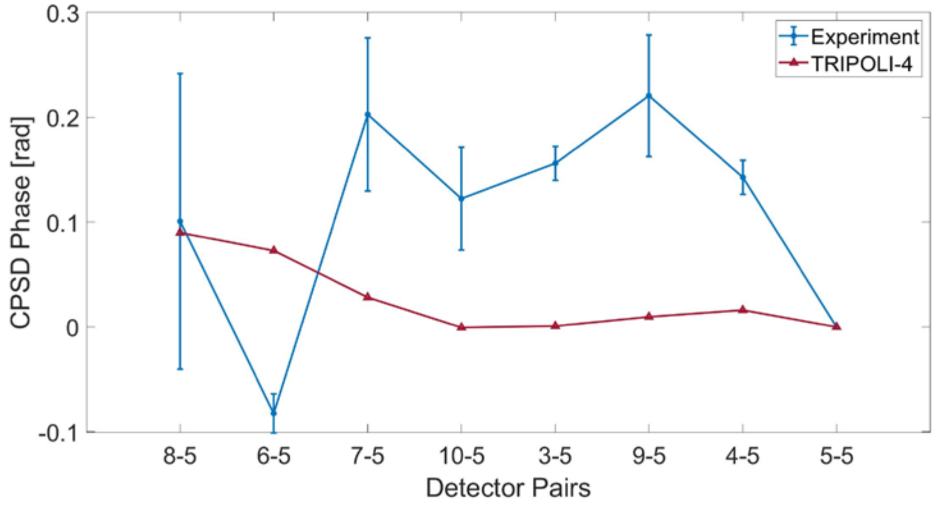
Convergence “issues” for noise induced by mechanical vibration

Experiment 13 – TRIPOLI

Amplitude



Phase



MCNP

Random walk process of the neutron noise calculation in MCNP

- First, a particle is emitted from the noise source position.
  - The energy, direction, weight of the particle are determined based on the noise source property. (this will be discussed later).
- The weight is generally complex. The weight carries two values: real and imaginary
- The distance to the next collision point is determined as usual: $s = -\frac{1}{\Sigma_t} \ln \xi$

$\xi =$ uniform pseudo random number between 0 and 1.

- But, we have to include the additional term during the course of the random walk. $-\frac{i\omega}{v(E)} \delta\phi(\boldsymbol{r}, \boldsymbol{\Omega}, E, \omega)$

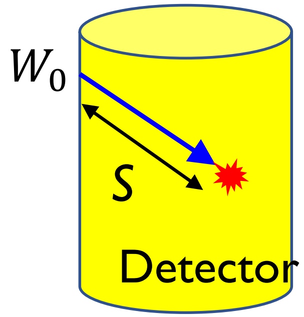
- Several methods are possible to include this term in the random walk process. MCNP adopts a "continuous absorption weighting" (CAW) method.

$$W_0 \xrightarrow{\quad s \quad} W_0 \exp\left(-\frac{i\omega s}{v}\right)$$

Every time a particle travels a distance, the weight continuously changes.

Detecting neutron noise in MCNP

- Track length estimator is chosen for neutron noise detection.
- Because the weight changes during the flight, the track length times weight is NOT given by $W_0 \cdot s$.
  - Instead, it is given by as follows:



$$TL(E) = \int_0^s W_0 \cdot exp\left(-\frac{i\omega}{v}s'\right) ds' = W_0 \frac{iv}{\omega}\left(exp\left(-\frac{i\omega}{v}s\right) - 1\right)$$

s = track length within a detector, *E* =energy of particle

Count of neutron noise is $C = TL(E) \cdot \Sigma_d(E)$

$\Sigma_d(E)$ Detector's macroscopic cross section

C is usually a complex value.

Noise source for COLIBRI

$$S(r,\Omega,E,\omega) \equiv -\delta\Sigma_t(r,E,\omega)\phi_0(r,\Omega,E)$$ → Total X-sec change

$$+\frac{\chi(E)}{4\pi\, k_{eff}}\iint \nu\delta\Sigma_f(r,E',\omega)\phi_0(r,\Omega',E')dE'd\Omega'$$ → Fission X-sec change

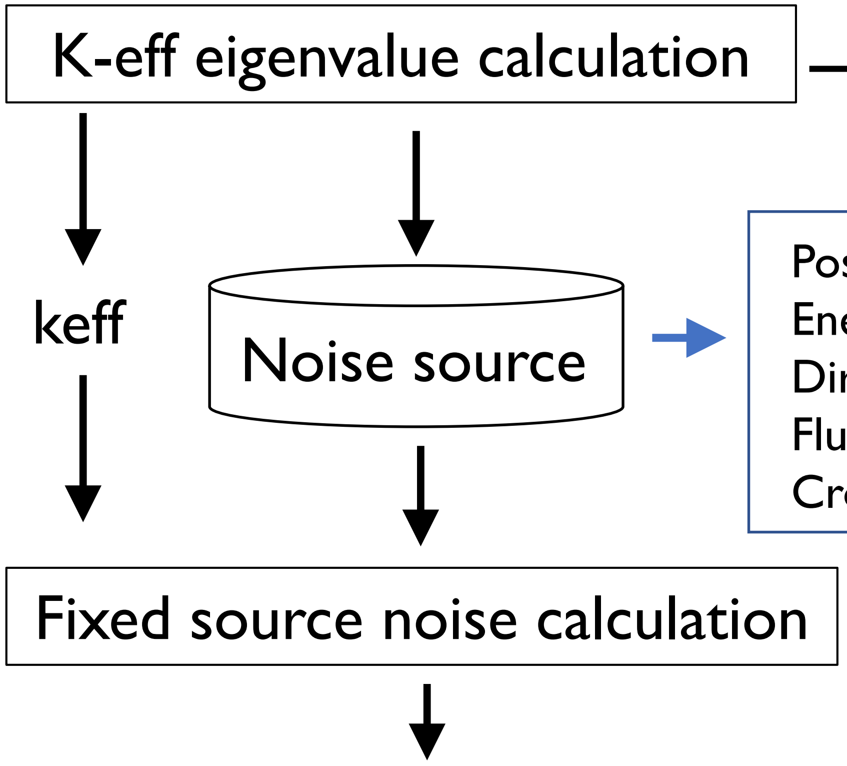
$$+\iint \delta\Sigma_s(r,E'\to E,\Omega'\to\Omega,\omega)\phi_0(r,\Omega',E')d\Omega'dE'$$ → Scattering X-sec change

$\phi_0(r,\Omega,E)$= Neutron flux in steady state

- The steady state neutron flux is in a critical state.
- Hence, before the neutron noise calculation, the *k*-effective eigenvalue calculation is performed to obtain $\phi_0(r,\Omega,E)$ and keff.

- For k-effective eigenvalue calculation, "kcode" option of MCNP is used.
- Modelling of the noise source is the same as TRIPOLI-4.

Overview of the neutron noise calculation



Steady state count rate of each detector for normalization of neutron noise

Contents of noise source file

- Position of collision: x, y, z
- Energy of the particle after collision: erg
- Direction of the particle after collision: u, v, w
- Flux(weight of the particle divided by total Xsec): $wgt/\Sigma_t(r,E)$ $=\phi_0(r,\Omega,E)$
- Cross section change $\delta\Sigma_s, \delta\Sigma_t, \delta\Sigma_f$

Detected Neutron noise

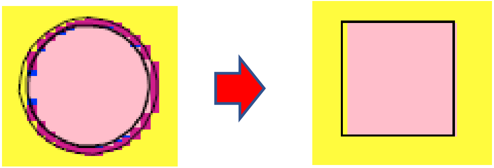
$$C = \int \Sigma_d(E)\delta\phi(E,\omega) = \sum_i \Sigma_d(E) TL_i(E)$$

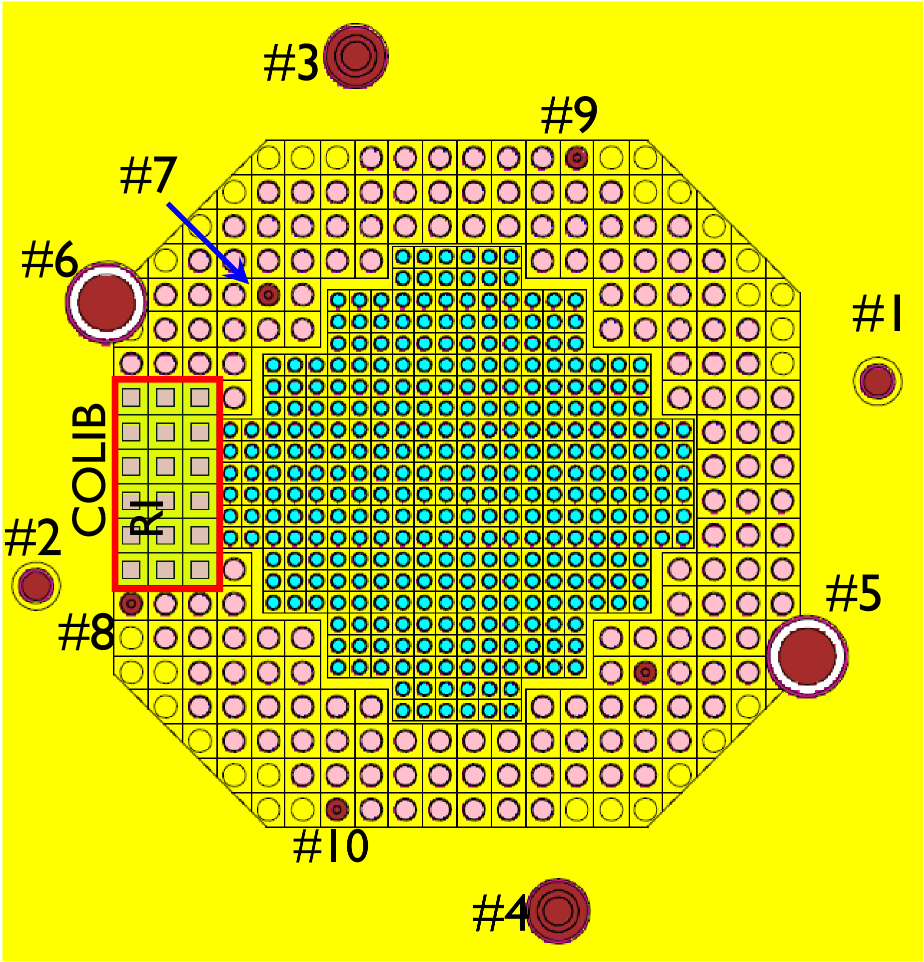
Acumulate the product of detector cross section and the track length for neutron noise

MCNP model for COLIBRI

For defining noise source, the vibrating fuel rods are approximated by square shape.

The claddings are replaced by the light-water moderator.





APSD and CPSD of MCNP

APSD and CPSD of each detector is normalized by the steady state count rate.

Normalized APSD= $\dfrac{C \cdot C^*}{C_0 \cdot C_0} = \dfrac{Re[C]^2 + Im[C]^2}{C_0^2}$ (*denotes complex conjugate)

C = count rate of noise (complex value)

C_0= steady state count rate by *k*-eff calculations (real value)

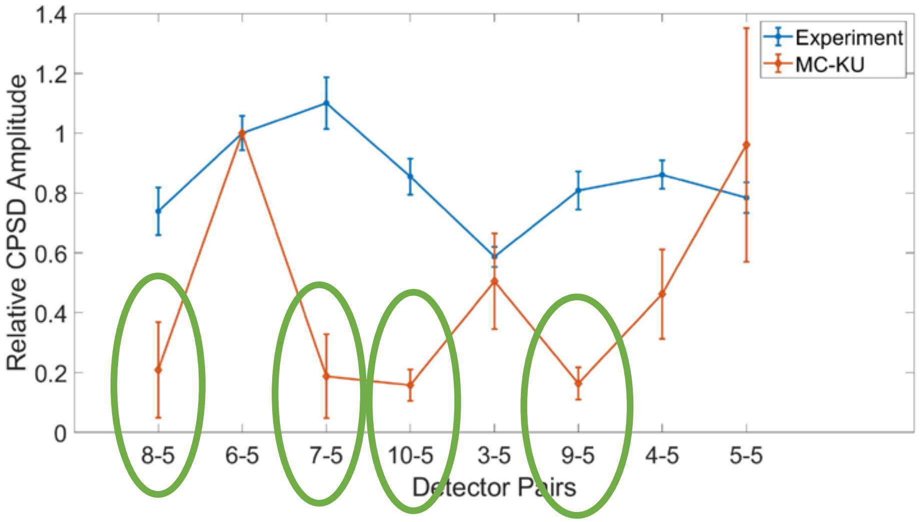
Normalized CPSD= $\dfrac{C_1 \cdot C_2^*}{C_{10} \cdot C_{20}}$ (*denotes complex conjugate)

C_i = count rate of noise of detector *i* (complex value)

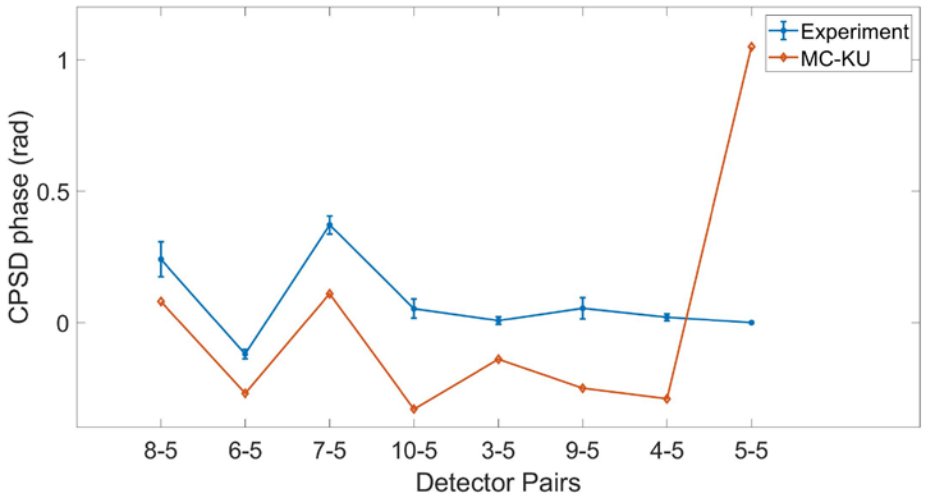
C_{i0} = steady state count rate of detector *i* by keff calculations (real value)

Experiment 12 – MCNP - KU

Amplitude



Phase



MCNP underestimates for $^{10}BF_3$ detectors.

Questions ?

Thank you