Monte Carlo Calculation method for Neutron Noise

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Kyoto University

Equation to be solved for neutron noise in the transport theory

$$\boldsymbol{\Omega} \cdot \nabla \delta\phi(\boldsymbol{r},\boldsymbol{\Omega},E,\omega) + \Sigma_{t0}(\boldsymbol{r},E)\delta\phi(\boldsymbol{r},\boldsymbol{\Omega},E,\omega)$$
$$= \int_{4\pi} d\boldsymbol{\Omega}' \int dE' \Sigma_{s0}(\boldsymbol{r},\boldsymbol{\Omega}' \to \boldsymbol{\Omega},E' \to E)\delta\phi(\boldsymbol{r},\boldsymbol{\Omega}',E',\omega)$$
$$+ \frac{\chi_p(E)}{4\pi k_{eff}} \int_{4\pi} d\boldsymbol{\Omega}' \int dE' \nu_p \Sigma_{f0}(\boldsymbol{r},E')\delta\phi(\boldsymbol{r},\boldsymbol{\Omega}',E',\omega)$$
$$+ \frac{\chi_d(E)}{4\pi k_{eff}} \int_{4\pi} d\boldsymbol{\Omega}' \int dE' \nu_{d,\omega} \Sigma_{f0}(\boldsymbol{r},E')\delta\phi(\boldsymbol{r},\boldsymbol{\Omega}',E',\omega)$$
$$- \frac{i\omega}{v(E)}\delta\phi(\boldsymbol{r},\boldsymbol{\Omega},E,\omega) + S(\boldsymbol{r},\boldsymbol{\Omega},E,\omega),$$

$\omega = angular\ frequency$

$i = \sqrt{-1}, v(E) = speed$

Only one delayed neutron group and only one fission nuclide are assumed.

- This transport equation is very similar to the ordinary fixed source neutron transport equation.

Differences from the ordinary transport equation

- The number of delayed neutrons is $\nu_{d,\omega} = \dfrac{\lambda^2 - i\omega\lambda}{\lambda^2 + \omega^2} \cdot \nu_d$

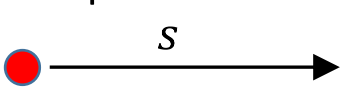
  This value is complex value and it is dependent of frequency.

- The fission spectra are divided by keff because the reactor is assumed to be critical.
- The biggest difference is the existence of $-\dfrac{i\omega}{v(E)}\delta\phi(\boldsymbol{r},\boldsymbol{\Omega},E,\omega)$

Random walk process of the neutron noise calculation

- First, a particle is emitted from the noise source position.
  - The energy, direction, weight of the particle are determined based on the noise source property. (this will be discussed later).
- The weight is generally complex. The weight carries two values: real and imaginary
- The distance to the next collision point is determined as usual: $s = -\frac{1}{\Sigma_t} \ln \xi$

  ξ = pseudo random number between 0 and 1.
- But, we have to include the additional term during the course of the random walk. $-\frac{i\omega}{v(E)} \delta\phi(\boldsymbol{r}, \boldsymbol{\Omega}, E, \omega)$
- Several methods are possible to include this term in the random walk process. MCNP adopts a “continuous absorption weighting” (CAW) method.

W_0  $W_0 \exp(-\frac{i\omega s}{v})$

Every time a particle travels a distance, the weight changed according to the equation above.

Process upon a collision event (1)

- There are two ways of how to treat the particle upon a collision. Implicit and Analog
- Implicit capture (default in MCNP)

  The particle weight is changed according to the absorption probability as:

$$W' = W \cdot \frac{\Sigma_s}{\Sigma_t} = \mathrm{Re}[W] \cdot \frac{\Sigma_s}{\Sigma_t} + i \cdot \mathrm{Im}[W] \cdot \frac{\Sigma_s}{\Sigma_t}$$

  - Russian Roulette

    After the weight reduction, Russian Roulette game is played for both Real and Imaginary parts. (Detail of Russian Roulette is omitted. Refer to Monte Carlo text book.)

    Only when both parts (real and imaginary) are killed at the same time, the particle is killed.
    If either of both parts survives, the particle's random walk still continues.

Process upon a collision event (2)

- Analog capture

  What reaction occurs is determined upon a collision event.

  If absorption (including fission) occurs,

  the particle is killed, and another new particle is started.

  If the reaction is not absorption,

  the particle survives and the weight is kept unchanged,

  the random walk continues.

- Implicit or Analog?

  If the frequency is higher or lower than a plateau frequency range, the implicit capture is often unavailable due to too many particles produced.

  The analogy capture method is stable for wide range of frequency.

  But, variance is not always small.

Process upon a collision event (3)

- Every time a collision occurs, the type of reaction is determined according to the probability of each reaction.
  - Capture (only analog capture), scattering, or fission
- In case of scattering and fission
  - The colliding particle is scattered as in normal Monte Carlo calculations.
  - The weight is reduced according to the absorption probability.
- In case of fission
  - Determine whether the fission neutron is prompt or delayed.

In case of **prompt** fission

- The number of prompt fission neutrons ν_p are stored in fission bank.
- The weight of the prompt fission neutrons : $W_p = W/k_{eff}$

W=particle weight before weight reduction

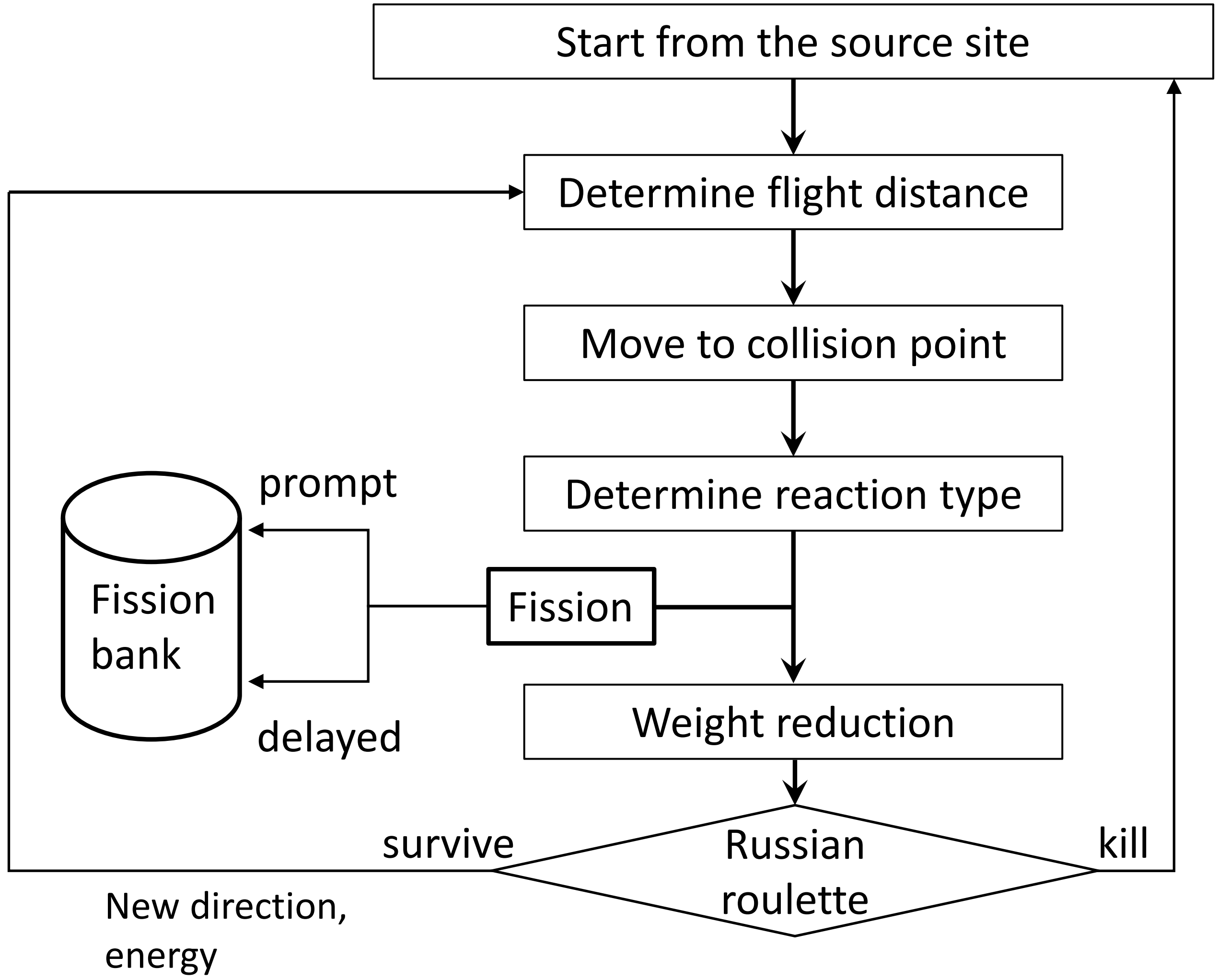
In case of **delayed** fission

- The number of delayed fission neutrons ν_d are stored in fission bank.
- The weight of the delayed fission neutrons :

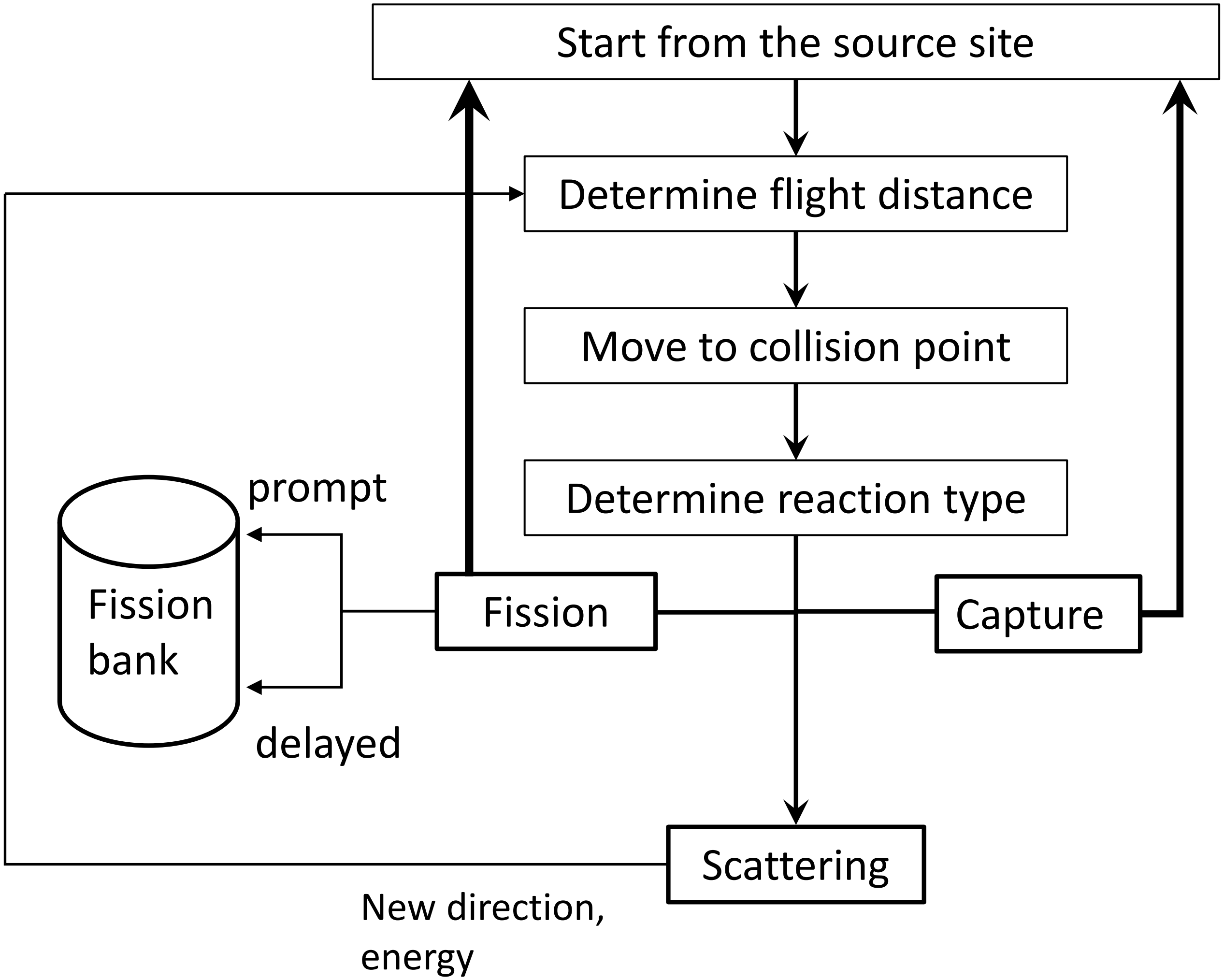
$$W_d = \frac{W}{k_{eff}} \frac{\lambda^2 - i\omega\lambda}{\lambda^2 + \omega^2}$$

- The fission neutrons in the fission bank will be emitted later.

Flow chart of MC noise calculation (implicit capture)

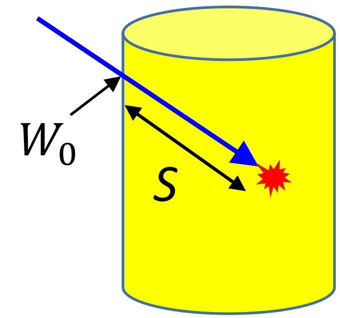


Flow chart of MC noise calculation (analog capture)



Counting neutron noise

- Two types of counting of neutron noise are possible. Collision and Track length estimators
- Track length estimator is chosen for neutron noise detection.
- Because the weight changes during the flight, the track length times weight is given by:



$$TL(E) = \int_0^s W_0 \cdot exp\left(-\frac{i\omega}{v}s'\right) ds' = W_0 \frac{iv}{\omega}\left(exp\left(-\frac{i\omega}{v}s\right) - 1\right)$$

S = track length within a detector, *E* =energy of particle

Count of neutron noise is $C = TL(E) \cdot \Sigma_d(E)$

$\Sigma_d(E)$ Detector macroscopic cross section

Of course, *C* is a complex value.

- When using track length estimator, the statistics do not depend on the density of the detecting material (e.g. B-10, U-235). It depends on the volume of the detector.

Noise source

$$S(r,\Omega,E,\omega) \equiv -\delta\Sigma_t(r,E,\omega)\phi_0(r,\Omega,E)$$
$$+\frac{\chi(E)}{4\pi k_{eff}}\iint \nu\delta\Sigma_f(r,E',\omega)\phi_0(r,\Omega',E')dE'd\Omega'$$
$$+\iint \delta\Sigma_s(r,E'\rightarrow E,\Omega'\rightarrow\Omega,\omega)\phi_0(r,\Omega',E')d\Omega'dE'$$

$\phi_0(r,\Omega,E)$= Neutron flux in steady state

k = integer

Time dependent cross section fluctuation is expanded as:

$$\delta\Sigma_x(r,t) = \sum_{k=-\infty}^{+\infty} c_k \exp(ik\omega_0 t)$$

ω_0 = angular frequency of vibration

The coefficient c_k is

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} \delta\Sigma_x(r,t)\exp(-ik\omega_0 t)\,dt$$

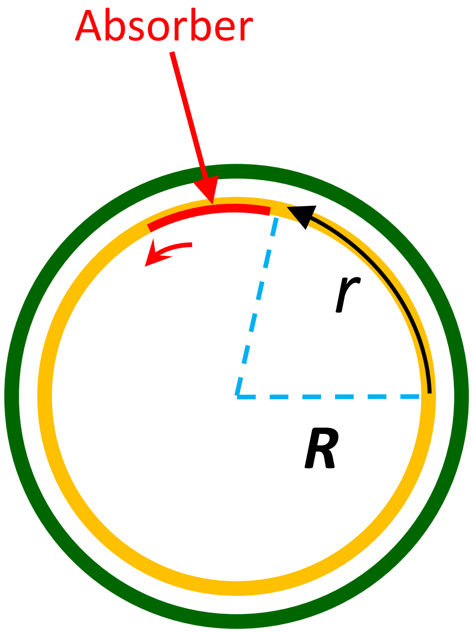
Fourier transform

$$\exp(ik\omega_0 t) \Rightarrow 2\pi\delta(\omega - k\omega_0)$$

$$\delta\Sigma_x(r,\omega) = 2\pi\sum_{k=-\infty}^{+\infty} c_k\delta(\omega - k\omega_0)$$

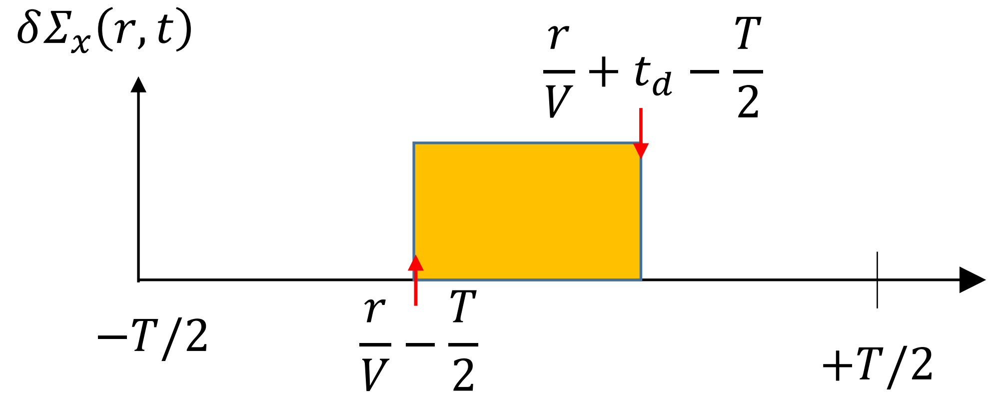
The noise source appears at the integer multiples of the vibration frequency.

Modeling of AKR-2 Rotating Absorber



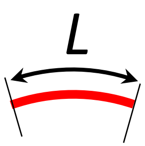
θ (rad)

$$0 < r < 2\pi R - L$$

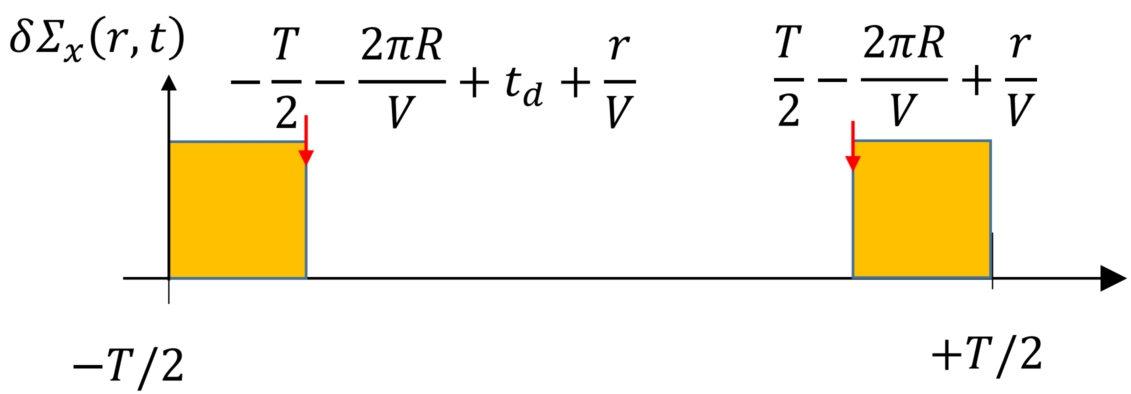


V = velocity of absorber = ωR

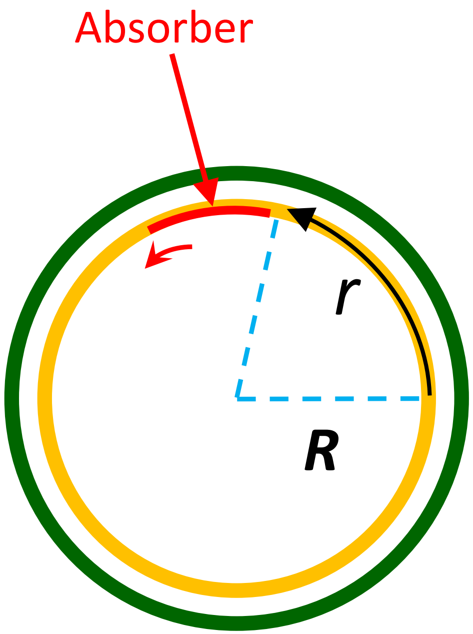
$r = R\theta, \quad t_d = L/V \qquad L$ = width of absorber = 2cm



$$2\pi R - L < r < 2\pi R$$



Modeling of AKR-2 Rotating Absorber



θ (rad)

No fission in the rotating absorber

$$S(r,\Omega,E,\omega) = -\delta\Sigma_t(r,E,\omega)\phi_0(r,\Omega,E) + \iint \delta\Sigma_s(r,E' \to E,\Omega' \to \Omega,\omega)\phi_0(r,\Omega',E')d\Omega' dE'$$

$$\delta\Sigma_x(r,\omega) = 2\pi \sum_{k=-\infty}^{+\infty} c_k \delta(\omega - k\omega_0)$$

$\omega_0 =$ angular frequency of vibration

$0 < r < 2\pi R - L$

$$c_k = \frac{\Delta}{2\pi k}[i \cdot cos(k\omega_0 t) + sin(k\omega_0 t)]_{t=a}^{t=b}$$

$$a = \frac{R\theta}{V} - \frac{T}{2}, \qquad b = \frac{R\theta}{V} + t_d - \frac{T}{2}$$

$\Delta = \Sigma_x - 0 = \Sigma_x$

Macroscopic X-section of Cd

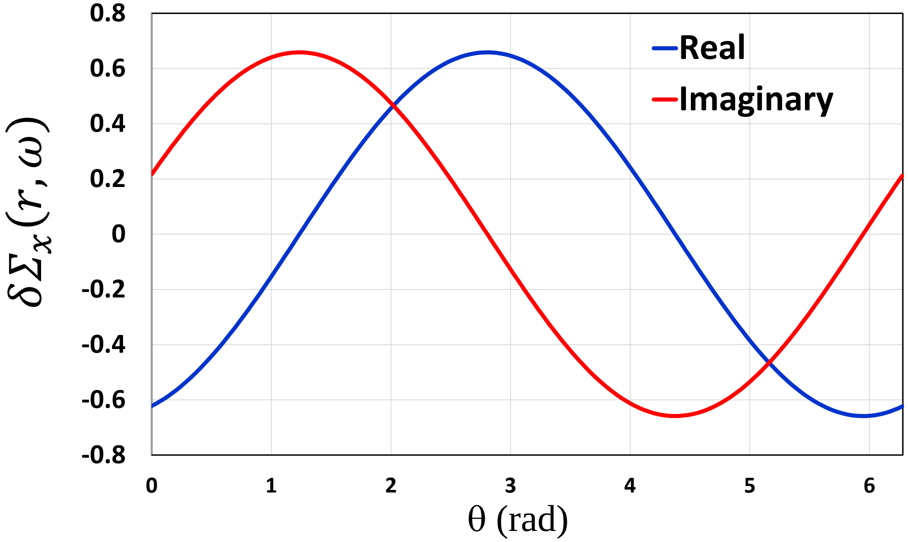
$2\pi R - L < r < 2\pi R$

$$c_k = \frac{\Delta}{2\pi k}\{[i \cdot cos(k\omega_0 t) + sin(k\omega_0 t)]_{t=a}^{t=b} + [i \cdot cos(k\omega_0 t) + sin(k\omega_0 t)]_{t=c}^{t=d}\}$$

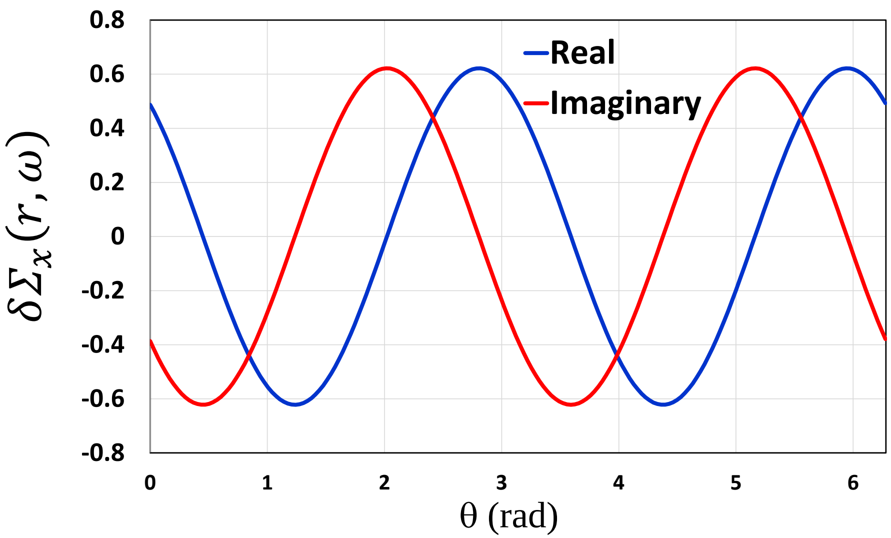
$$a = -\frac{T}{2} \qquad b = \frac{r}{V} - \frac{2\pi R}{V} + t_d - \frac{T}{2} \qquad c = \frac{T}{2} - \frac{2\pi R}{V} + \frac{r}{V} \qquad d = \frac{T}{2}$$

Noise source of rotation frequency 1 Hz

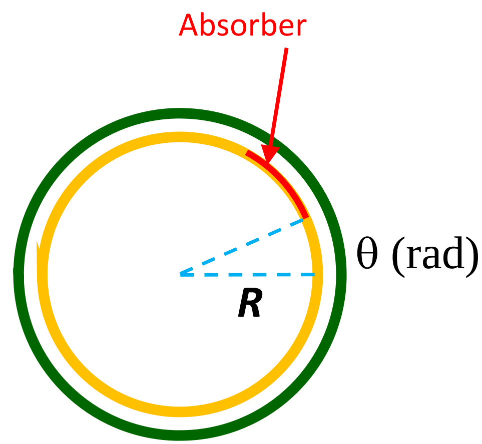
$k = 1, \quad 1\text{ Hz}$



$k = 2, \quad 2\text{Hz}$

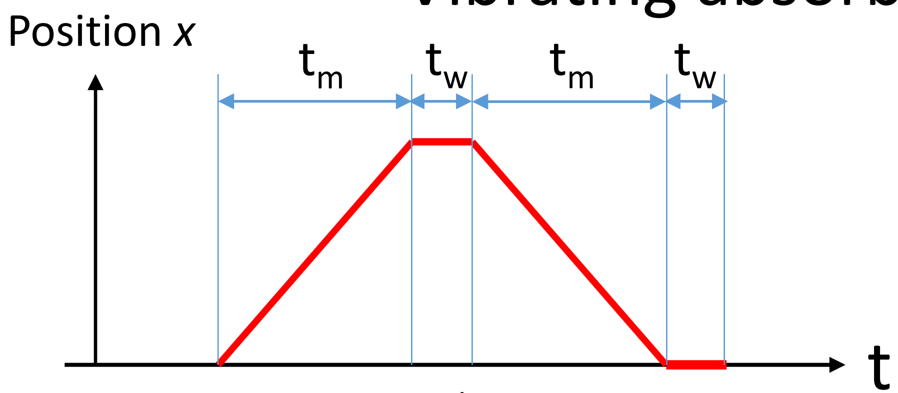


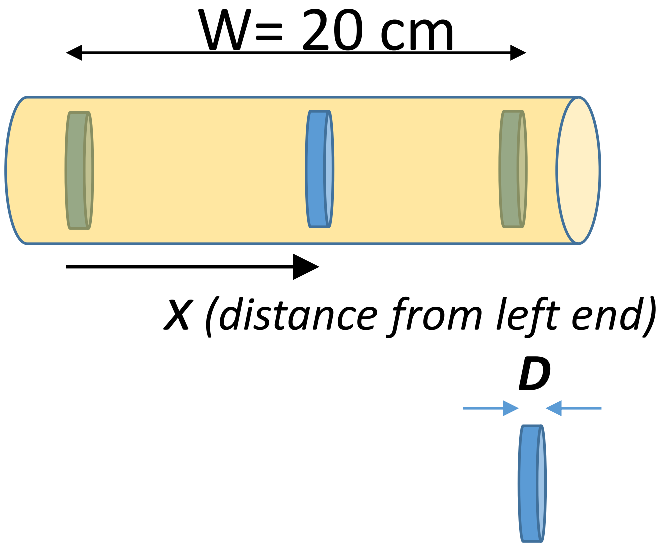
$$\delta\Sigma_x(r,\omega) = 2\pi \sum_{k=-\infty}^{+\infty} c_k \delta(\omega - k\omega_0)$$



- If the flux distribution is almost flat regardless of θ, the neutron noise source is cancelled.
- The flat flux distribution results in small neutron noise.

Vibrating absorber





$$\delta\Sigma_x(r,\omega) = 2\pi \sum_{k=-\infty}^{+\infty} c_k \delta(\omega - k\omega_0)$$

$0 < x < D$ Left end of vibration region

V=velocity of absorber

$$c_k = \frac{\Delta}{2\pi k}[i \cdot cos(k\omega_0 t) + sin(k\omega_0 t)]_{t=a}^{t=b} \quad a = -x/V, \quad b = t_w + x/V$$

$D < x < W - D$

$$c_k = \frac{\Delta}{2\pi k}\{[i \cdot cos(k\omega_0 t) + sin(k\omega_0 t)]_{t=a}^{t=b} + [i \cdot cos(k\omega_0 t) + sin(k\omega_0 t)]_{t=c}^{t=d}\}$$

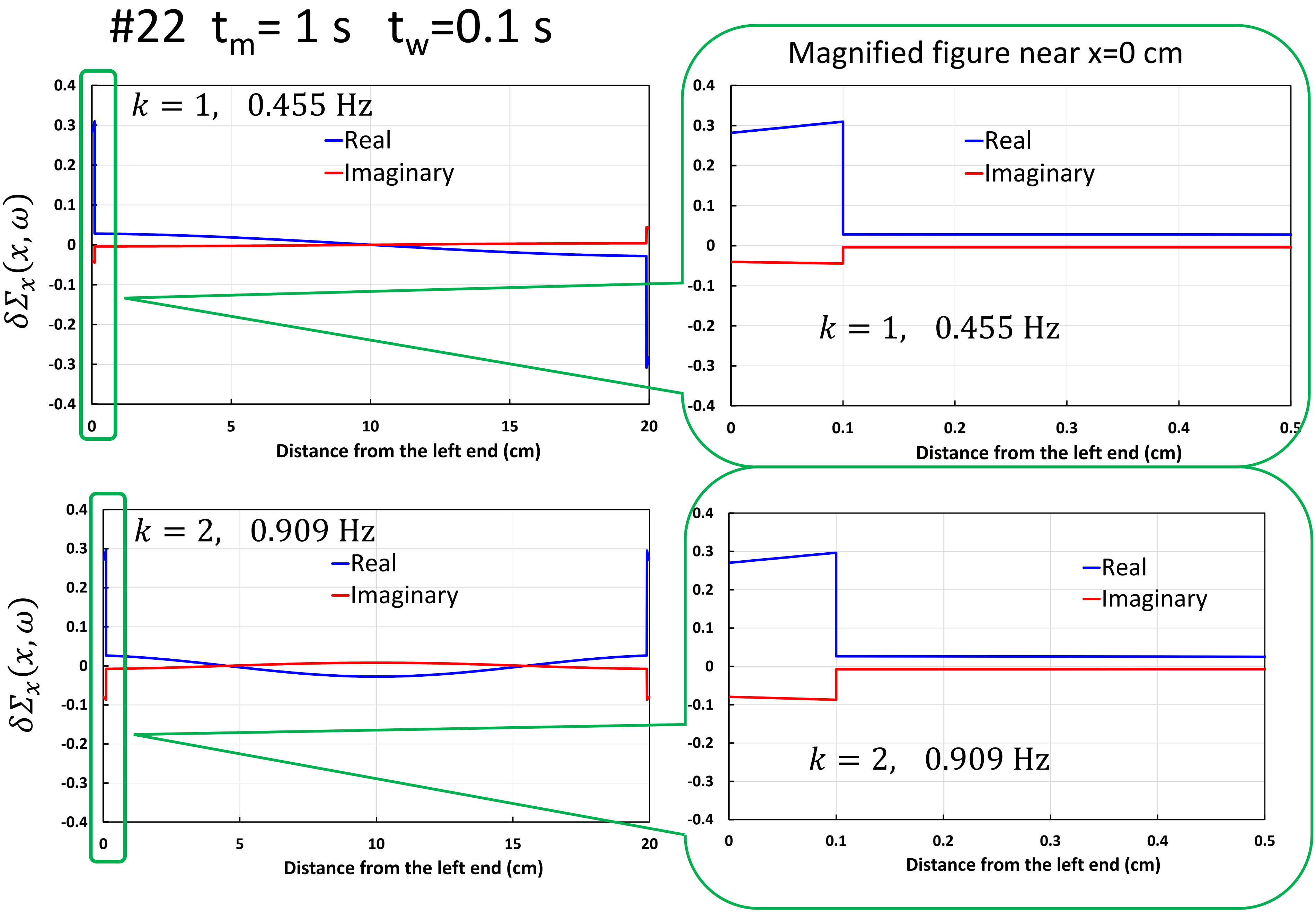
$$a = -x/V, \quad b = D/V - x/V, \quad c = t_w + x/V - D/V, \quad d = t_w + x/V$$

$W - D < x < W$ Right end of vibration region

$$c_k = \frac{\Delta}{2\pi k}\{[i \cdot cos(k\omega_0 t) + sin(k\omega_0 t)]_{t=a}^{t=b} + [i \cdot cos(k\omega_0 t) + sin(k\omega_0 t)]_{t=c}^{t=d}\}$$

$$a = -T/2, \quad b = W/V - x/V - t_m, \quad c = x/V - D/V + t_w, \quad d = T/2$$

Noise source of vibration frequency #22 (f=0.455 Hz)



Flow of Monte Carlo calculation for neutron noise

- Before starting the transport calculation for neutron noise (fixed source calculation), we need to obtain the neutron noise source.

$$S(r,\Omega,E,\omega) = -\delta\Sigma_t(r,E,\omega)\phi_0(r,\Omega,E) + \iint \delta\Sigma_s(r,E' \to E,\Omega' \to \Omega,\omega)\phi_0(r,\Omega',E')d\Omega' dE'$$

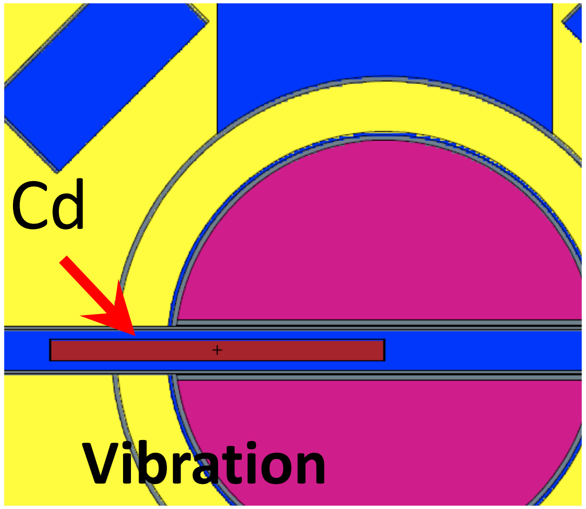
- We need to know the neutron flux in the steady state. $\phi_0(r,\Omega,E)$

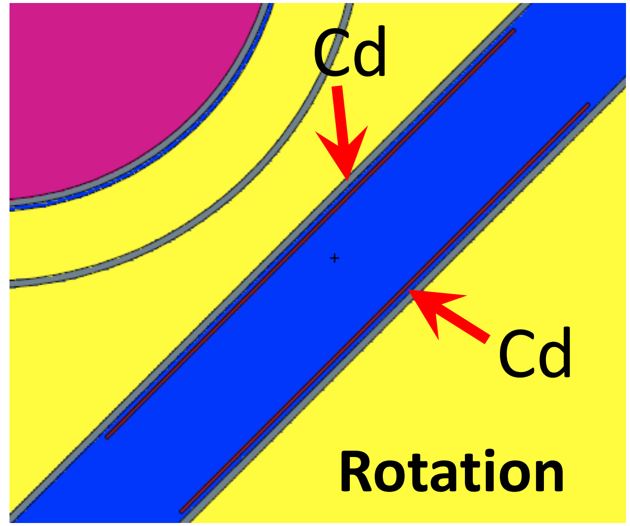
- The steady state neutron flux is in a critical state.
- Hence, before the neutron noise calculation, the k-effective eigenvalue calculation has to be performed to obtain $\phi_0(r,\Omega,E)$ and keff.

- For k-effective eigenvalue calculation, "kcode" option of MCNP is used.

How to perform k-effective eigenvalue calculation for noise source

- Neutron absorber Cd is allocated to the ***whole*** region where the absorber sweeps.





- Each time a neutron collides with Cd, the following information are stored.

For noise source due to total Xsec. Change: $-\delta\Sigma_t(r, E, \omega)\phi_0(r, \Omega, E)$

Position of collision: x, y, z
Energy of the colliding particle: erg
Direction of the colliding particle: u, v, w
Flux(weight of the particle divided by total Xsec): $wgt/\Sigma_t(r, E)$ $=\phi_0(r, \Omega, E)$
Macroscopic total cross section: $\Sigma_t(r, E)$
Cross section change: $\delta\Sigma_t(r, E) = \Sigma_t(r, E) - 0 = \Sigma_t(r, E)$

How to perform k-effective eigenvalue calculation for noise source

- Each time a neutron collides with Cd, the following information are stored.

For noise source due to scattering Xsec. Change: $\iint \delta\Sigma_s(r, E' \to E, \Omega' \to \Omega, \omega)\phi_0(r, \Omega', E')d\Omega' dE'$

Position of collision: x, y, z
Energy of the particle after collision: erg
Direction of the particle after collision: u, v, w
Flux(weight of the particle divided by total Xsec): $wgt/\Sigma_t(r, E)$ =$\phi_0(r, \Omega, E)$
Cross section change $\delta\Sigma_s(r, E) = \Sigma_s(r, E) - 0 = \Sigma_s(r, E)$

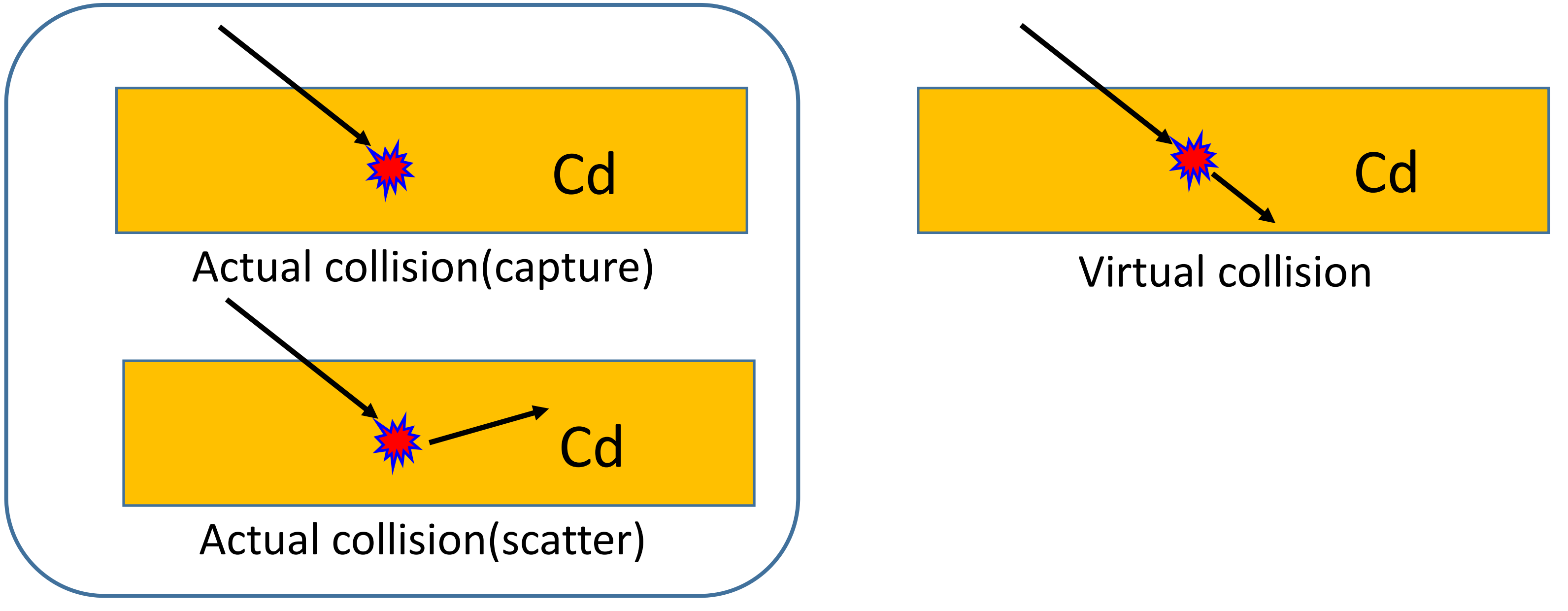
- In case of Cd absorber, $\Sigma_t \gg \Sigma_s$.

$$-\delta\Sigma_t(r, E, \omega)\phi_0(r, \Omega, E) \gg \iint \delta\Sigma_s(r, E' \to E, \Omega' \to \Omega, \omega)\phi_0(r, \Omega', E')d\Omega' dE'$$

The noise source is mostly dominated by the change of the total cross section, and the effect of the scattering cross section is minor.

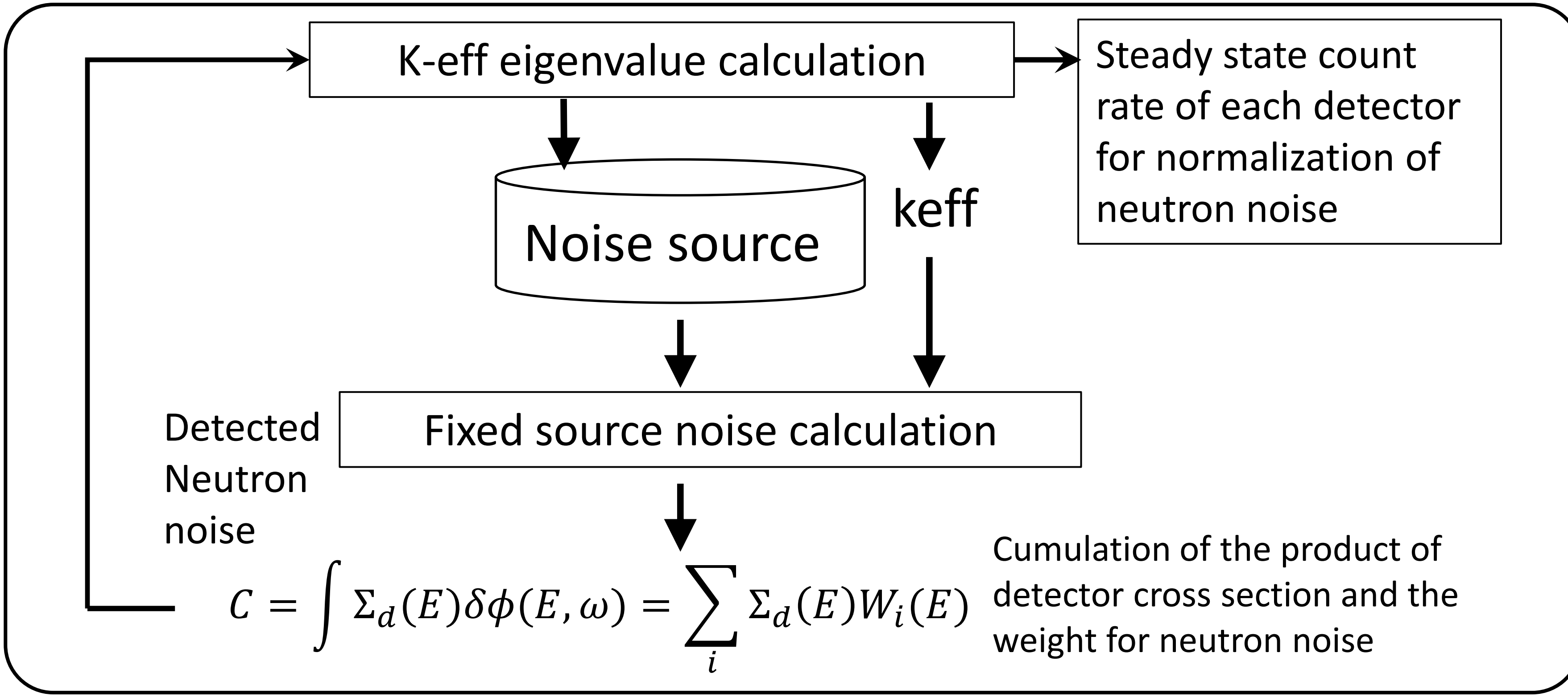
How to perform k-effective eigenvalue calculation for noise source

- The collision at the Cd absorber is treated as a virtual collision.
- A thermal neutron that collides with Cd is mostly absorbed and occasionally scattered
- In the virtual collision, the neutron keeps flying after the collision without changing anything (weight, energy, direction). But the noise source is scored.



- The function of the virtual collision is not equipped with the original MCNP.
- MCNP code is modified so that the virtual collision can be treated in a specified cell.

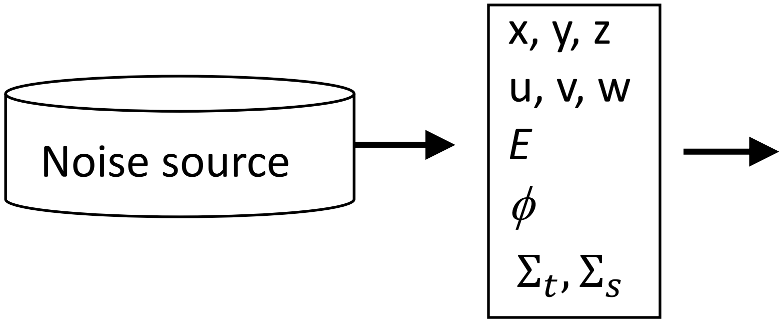
Overview of the neutron noise calculation



- A huge storage is required for the noise source file in order to obtain the neutron noise with less statistical uncertainty.
- To reduce the file size, each calculation is performed with a moderate number of source particles.
- One calculation is not enough to obtain statistically significant results.
- Therefore, the calculation flow above is repeated many times to finally obtain the neutron noise.

Starting noise source in fixed source noise calculation

- A noise source is started from the noise source position (Cd absorber).
- The position, energy, flux, total or scattering cross sections are read from the noise source file.



Weight of starting particle

$$W = c_k \phi \qquad k = 1, 2, 3, \cdots$$

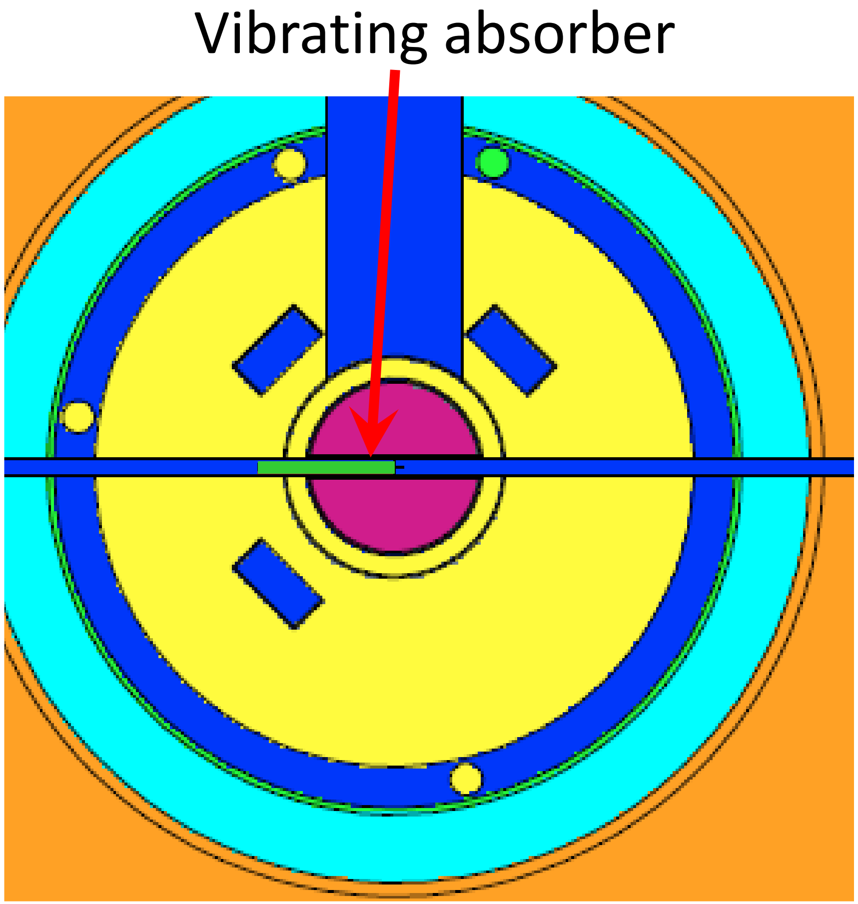
For example, c_k is given for the rotating absorber

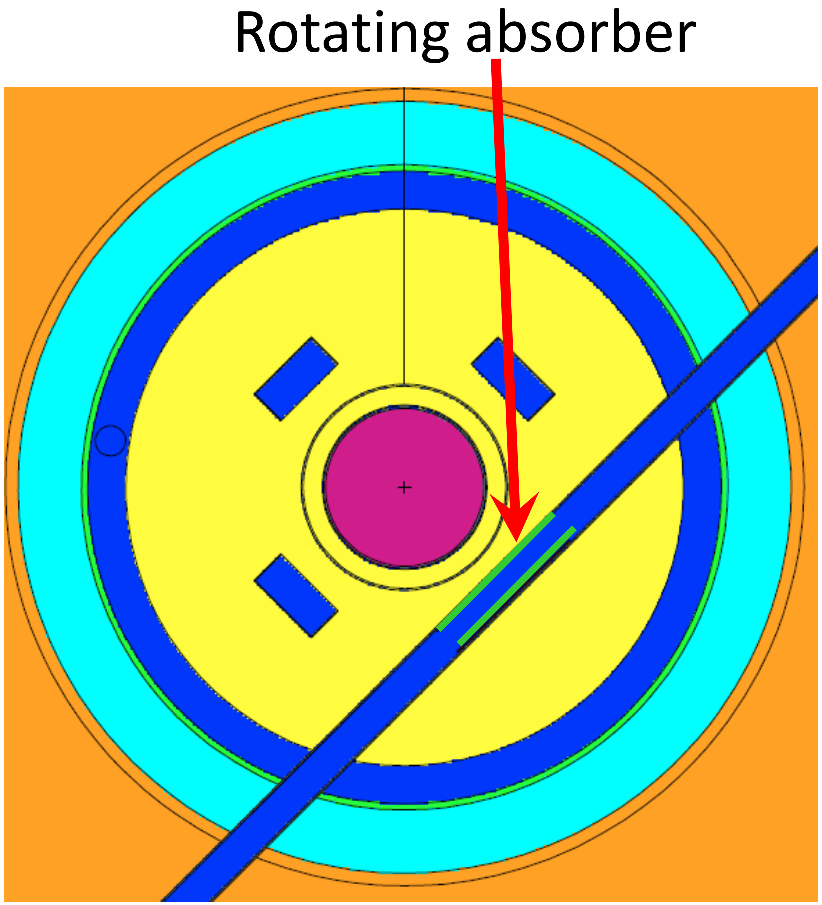
Total cross section change
$$c_k = \frac{-\Sigma_t}{k}\left[i \cdot cos(k\omega_0 t) + si\,n(\,k\omega_0 t)\right]_{t=a}^{t=b}$$

Scattering cross section change
$$c_k = \frac{\Sigma_s}{k}\left[i \cdot cos(k\omega_0 t) + si\,n(\,k\omega_0 t)\right]_{t=a}^{t=b}$$

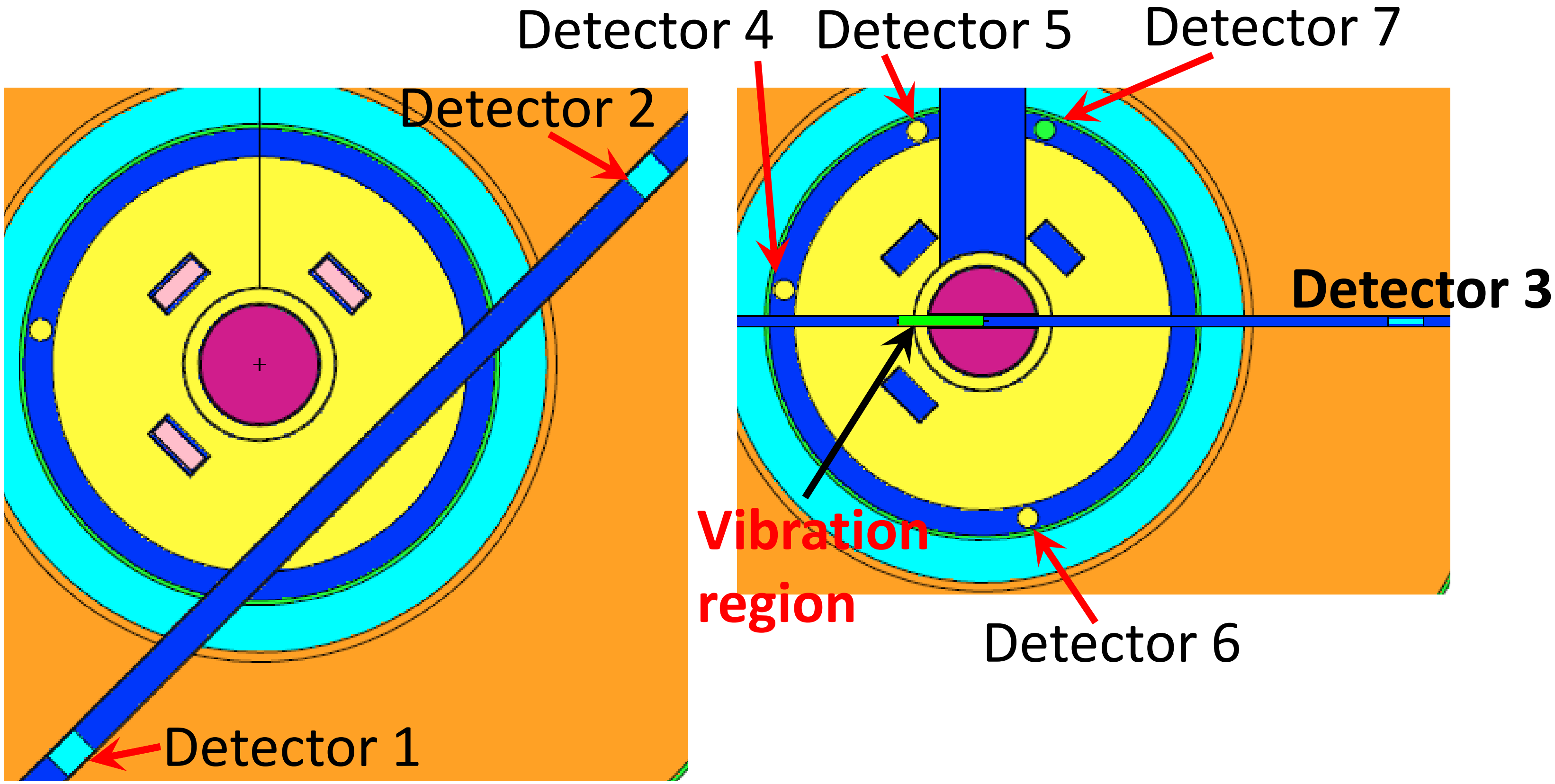
As k increases, c_k and the noise source become smaller.

Location of the noise source





Detector Arrangement of AKR-2



The volume of Detector 3 is small and it is far from the fuel.

The count in Detector 3 is low and the statistical uncertainty is not good.

APSD and CPSD

APSD and CPSD of each detector is normalized by the steady state count rate.

Normalized APSD= $\dfrac{C \cdot C^*}{C_0 \cdot C_0} = \dfrac{Re[C]^2 + Im[C]^2}{C_0^2}$ (*denotes complex conjugate)

C= count rate of noise (complex value)

C_0= stead state count rate by k-eff calculations (real value)

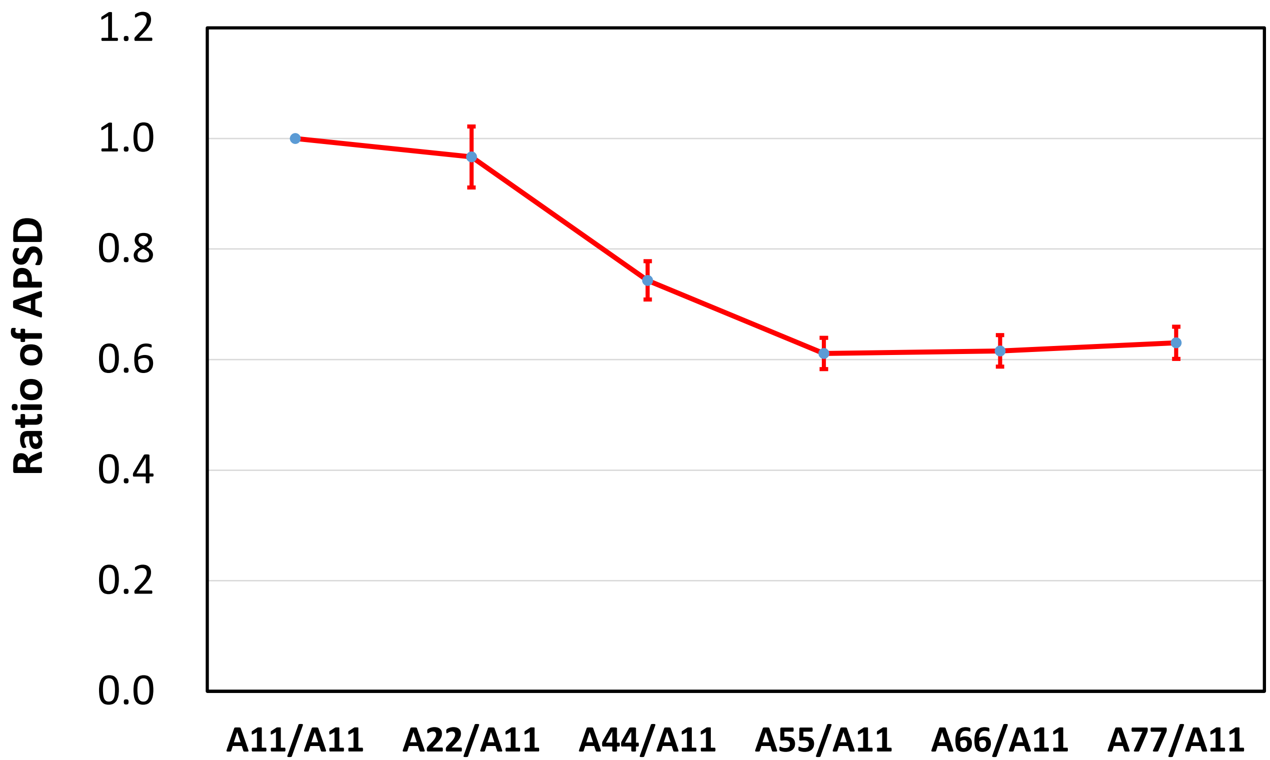
Normalized CPSD= $\dfrac{C_1 \cdot C_2^*}{C_{10} \cdot C_{20}}$ (*denotes complex conjugate)

C_i= count rate of noise of detector *i* (complex value)

C_{i0}= stead state count rate of detector *i* by keff calculations (real value)

Relative APSD (Vibrating Absorber) #22

f= 0.455 Hz and *k*=1 APSD with respect to Detector 1



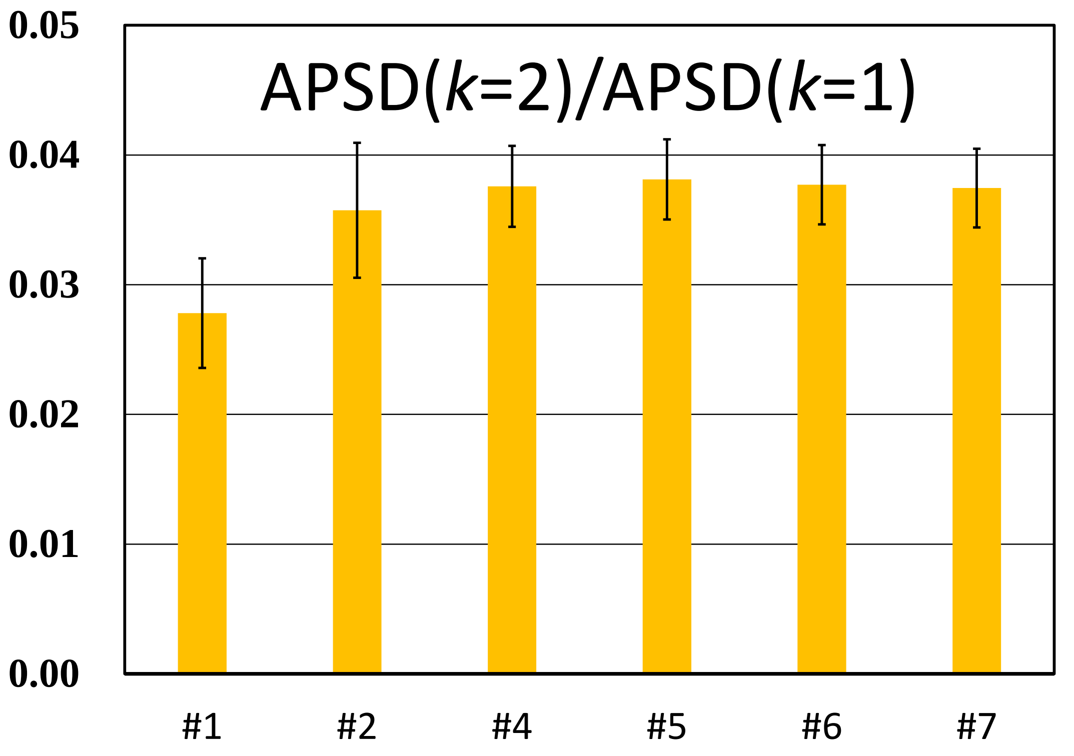
- Detector 3 is omitted here because the uncertainty is large.
- The gradient of flux is relatively large in the vibration region.
- The neutron noise can be calculated with less statistical uncertainty.

Relative APSD (Vibrating Absorber) #22

f_0= 0.455 Hz

The same calculations were performed for *k*=2 ($f=2*f_0$=0.91 Hz)

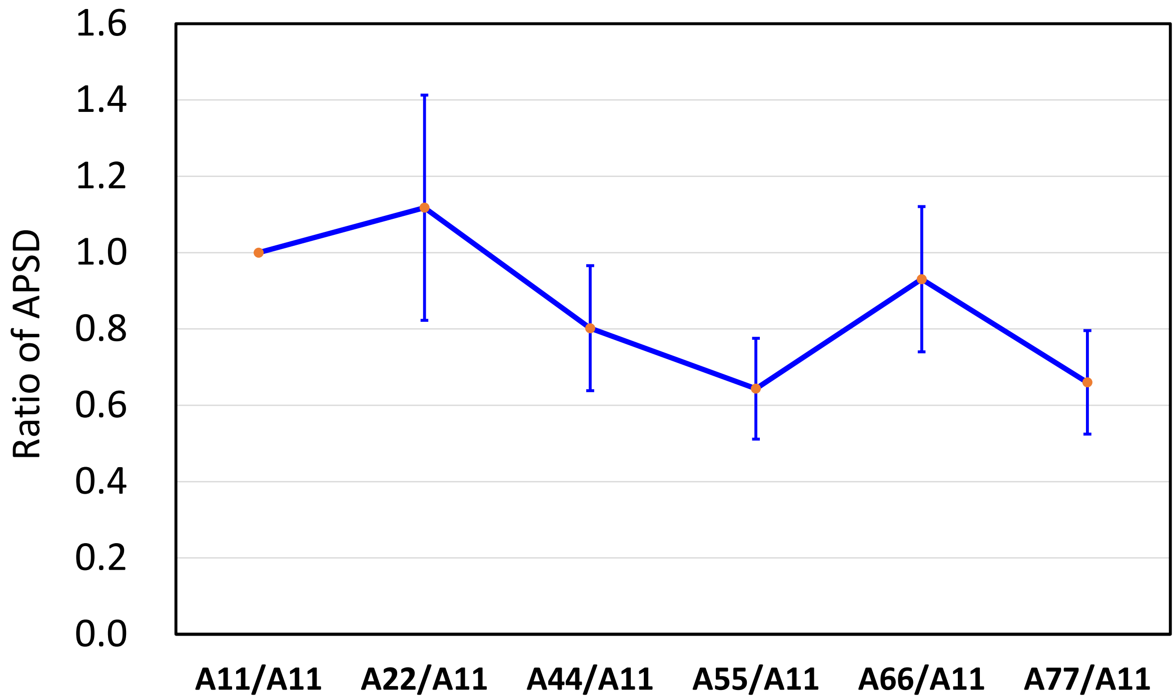
(APSD at 2nd freq. (*k*=2) with respect to vibration freq.(*k*=1))



- APSD at *k*=2 is about 1/25 of the vibration frequency (*k*=1).

Relative APSD (**Rotating** Absorber) #9

f= 1 Hz APSD with respect to Detector 1

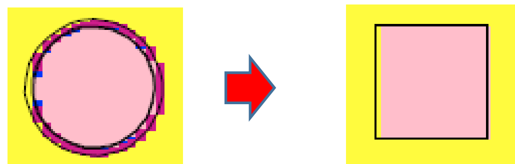


- The gradient of flux is not large enough in the rotation region.
- The statistical uncertainty is large in spite of spending long computation time. But much better than COLIBRI experiment.
- The neutrons that react with the rotating absorber is the neutrons escaping from the fuel.
- The noise source **rarely** contributes to fission chain reaction. That is why the neutron noise produced by the rotating absorber is small and the uncertainty is large.

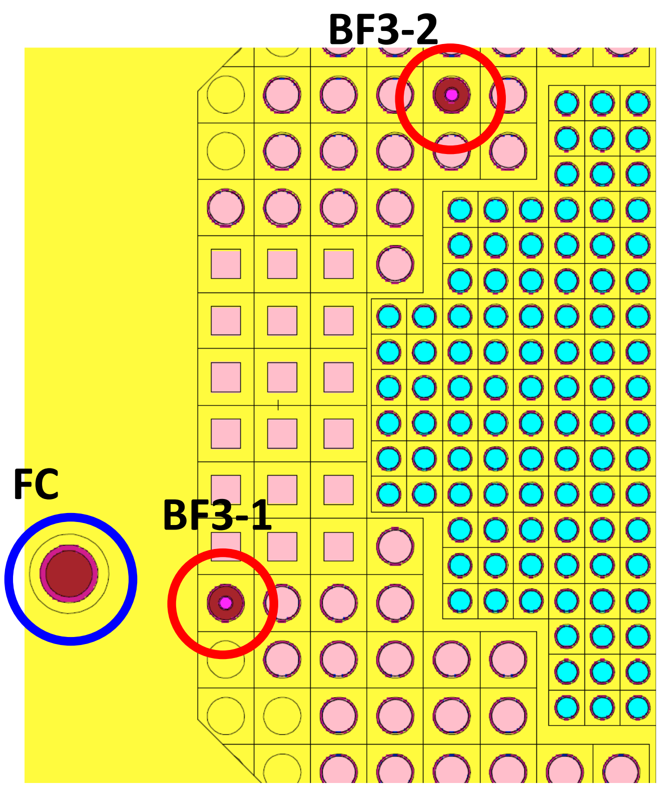
Recommendation Detector 3 and the rotating absorber should be brought close to the fuel.

Task 2.2 MCNP calculations for COLIBRI experiment in CROCUS

For defining noise source, the vibrating fuel rods are approximated by square shape. The claddings are replaced by the light-water moderator.



The noise sources are started from the 18 vibrating fuel rods.



The particles that represent the neutron noise are detected by two BF_3 counter and one fission chamber.

(**FC** is not used for the noise measurement, but it was chosen for the calculation due to the proximity to the noise source)

Noise sources are calculated according to the definition of the equation below

$$S(r,\Omega,E,\omega) = -\delta\Sigma_t(r,E,\omega)\phi_0(r,\Omega,E) + \frac{\chi(E)}{4\pi k_{eff}}\iint \nu\delta\Sigma_f(r,E',\omega)\phi_0(r,\Omega',E')dE'd\Omega' + \iint \delta\Sigma_s(r,E'\to E,\Omega'\to\Omega,\omega)\phi_0(r,\Omega',E')d\Omega'dE'$$

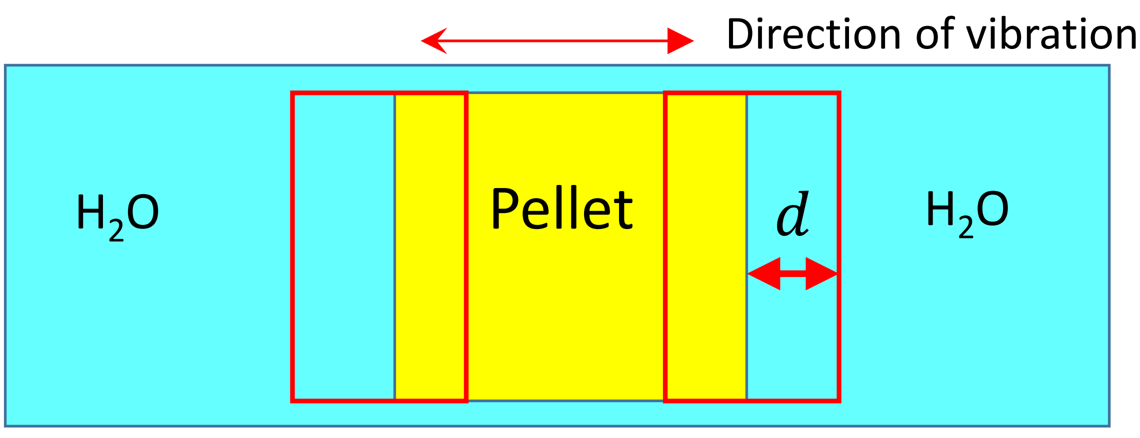
$\phi_0(r,\Omega,E)$: steady state neutron flux, which is calculated by the criticality calculation for the critical state.

$$\delta\Sigma(r,E,\omega) = -2(\Sigma_F(E) - \Sigma_{H2O}(E))\frac{\sin(2m\omega_0\tau(r))}{2m} \qquad \omega = 2\omega_0, 4\omega_0, 6\omega_0,\cdots$$

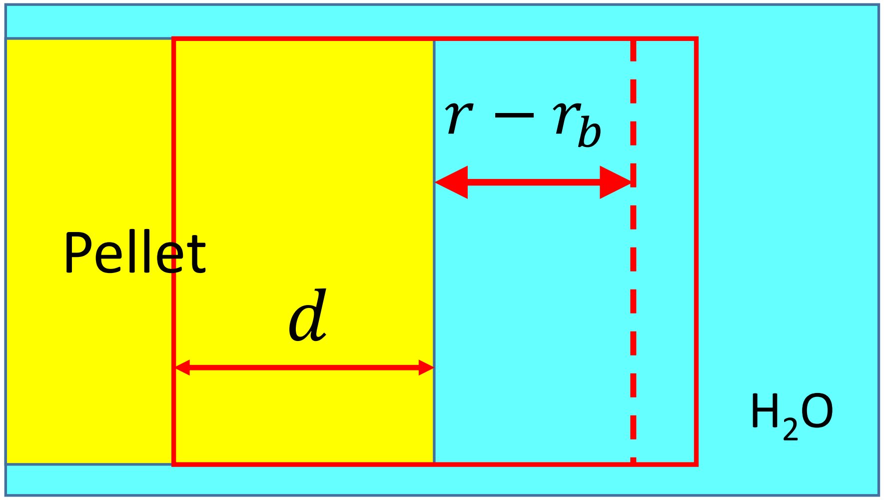
$$\delta\Sigma(r,E,\omega) = -2i(\Sigma_F(E) - \Sigma_{H2O}(E))\frac{\cos((2m+1)\omega_0\tau(r))}{2m+1} \qquad \omega = \omega_0, 3\omega_0, 5\omega_0,\cdots$$

$$\tau(r) = \sin^{-1}\left(\frac{r-r_b}{d}\right)/\omega_0$$

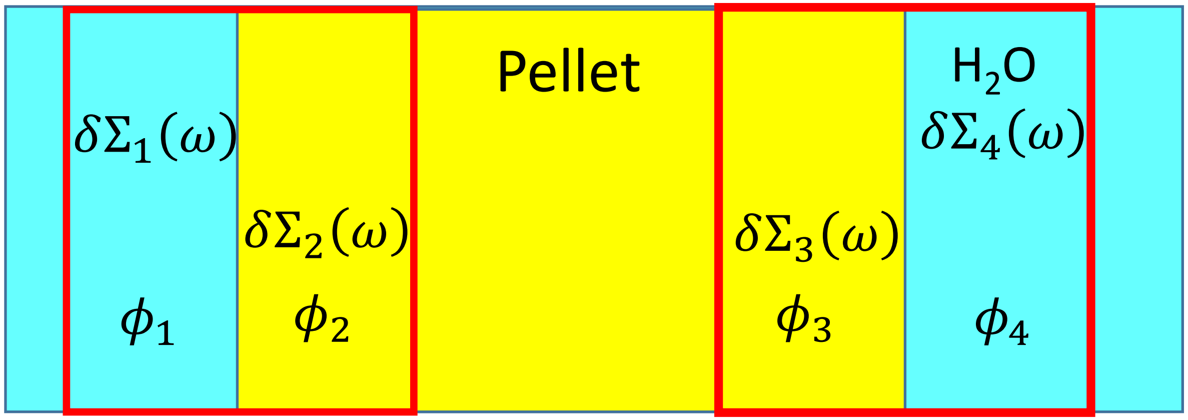
A. Rouchon, R. Sanchez, ICAPP 2015



d amplitude of vibration



$$\tau(r) = \sin^{-1}\left(\frac{r - r_b}{d}\right)/\omega_0$$



$\omega = 2\omega_0, 4\omega_0, 6\omega_0, \cdots \quad \delta\Sigma_3(\omega) = -\delta\Sigma_4(\omega) = \delta\Sigma_2(\omega) = \delta\Sigma_1(\omega)$

$\omega = \omega_0, 3\omega_0, 5\omega_0, \cdots \quad \delta\Sigma_3(\omega) = \delta\Sigma_4(\omega) = -\delta\Sigma_2(\omega) = -\delta\Sigma_1(\omega)$

Eqs.(23)&(24) in ICAPP paper

$\omega = \omega_0, 3\omega_0, 5\omega_0, \cdots \quad S(\omega) \approx \delta\Sigma(\omega)((\phi_3 + \phi_4) - (\phi_1 + \phi_2)) \approx 0$

The noise is very small for the frequency of the vibration frequency.
A huge computation time is needed to obtain statistically significant results.

$\omega = 2\omega_0, 4\omega_0, 6\omega_0, \cdots \quad S(\omega) \approx \delta\Sigma(\omega)((\phi_2 + \phi_3) - (\phi_1 + \phi_4)) \gg 0$

The noise is relatively large for the frequency of the even number of the vibration frequency.

In MCNP calculations, the steady state flux of each region (pellet, water) is used for $\phi_1, \phi_2, \phi_3, \phi_4$. How about TRIPOLI4?

The time average of $(\phi_1, \phi_2,)$ and (ϕ_3, ϕ_4) is more reasonable for the steady state flux?

Conclusion

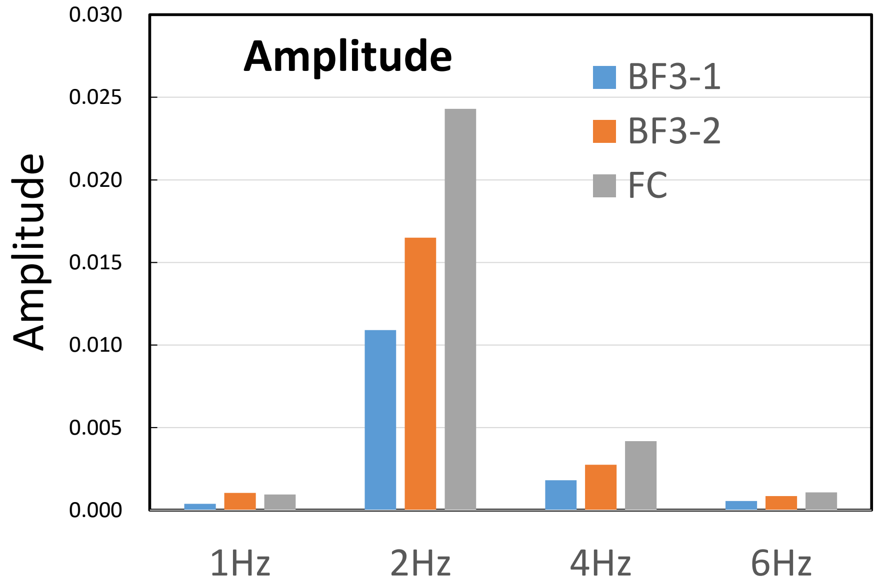
According to the definition of noise source for vibrating fuel rods,

- The noise source is very small for vibration frequency (1 Hz) and the odd number of the vibration frequency (3 Hz, 5 Hz …).
- In order to obtain statistically significant results, Monte Carlo calculations require very huge number of histories for 1 Hz, 3Hz , 5 Hz, ….

- If ω_0 = 1Hz, the noise source and the resulting neutron noise are relatively large for 2 Hz, 4 Hz, 6 Hz, ….
- For even number of the vibration frequency, Monte Carlo calculations can produce the statistically significant results by acceptable computation time.

- Is this phenomenon unphysical?
- The intensity of noise source depends on how to define the steady state flux ϕ_0.
- In this model, ϕ_0 is the flux in the unperturbed geometry.

Vibration frequency = 1Hz, ±1.5 mm



It was hard to obtain converged results for 3 Hz, 5Hz, ….
Thus, they are omitted in this figure.