DE LA RECHERCHE À L'INDUSTRIE

Advances on the noise model in the frequency domain

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CORTEX

Core monitoring techniques and experimental validation and demonstration

- TRIPOLI-4 solves the (complex) noise equation in the frequency domain:

$$\left[\frac{i\omega}{v} + \mathbf{\Omega}\cdot\nabla + \Sigma_{t,0} - S_0 - F_\omega\right]\delta\varphi = Q_\omega$$

- Decompose the noise δφ and the source Q_ω into real and imaginary parts:

$$\delta\varphi = \{\delta\mathcal{R}, \delta\mathcal{I}\}$$
$$Q_\omega = \{Q_\mathcal{R}, Q_\mathcal{I}\}$$

- We have the **coupled system** of transport equations

$$\begin{bmatrix} L^* & -P \\ P & L^* \end{bmatrix} \begin{bmatrix} \delta\mathcal{R} \\ \delta\mathcal{I} \end{bmatrix} = \begin{bmatrix} Q_\mathcal{R} \\ Q_\mathcal{I} \end{bmatrix}$$

- Where: $L^* = \mathbf{\Omega}\cdot\nabla + \Sigma_{t,0} - S_0 - F_p - \sum_j \frac{\lambda_j^2}{\lambda_j^2+\omega^2} F_d^j$

$$P = \frac{\omega}{v} + \sum_j \frac{\lambda_j\omega}{\lambda_j^2+\omega^2} F_d^j$$

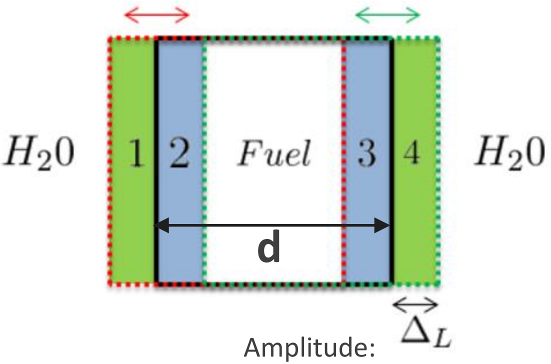
Sampling the noise source Q_ω: $-\delta L(r,t)\Psi_0(r)$

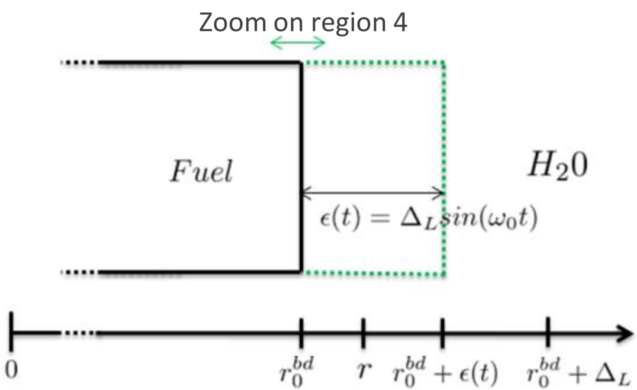
$$
\begin{aligned}
&-\delta L(\omega)\psi_0 = \\
&-\frac{\delta\Sigma_t(\mathbf{r},E,\omega)}{\Sigma_t(\mathbf{r},E)}\Sigma_t(\mathbf{r},E)\psi_0(\mathbf{r},\mathbf{\Omega},E) \\
&+\iint f_s(\mathbf{\Omega}'\cdot\mathbf{\Omega},E'\to E)\frac{\delta\Sigma_s(\mathbf{r},E',\omega)}{\Sigma_s(\mathbf{r},E')}\Sigma_s(\mathbf{r},E')\psi_0(\mathbf{r},\mathbf{\Omega}',E')dE'd\mathbf{\Omega}' \\
&+\frac{\chi_p(E)}{4\pi}\iint \nu_p(E')\frac{\delta\Sigma_f(\mathbf{r},E',\omega)}{\Sigma_f(\mathbf{r},E')}\Sigma_f(\mathbf{r},E')\psi_0(\mathbf{r},\mathbf{\Omega}',E')dE'd\mathbf{\Omega}' \\
&+\sum_j \frac{\lambda_j}{\lambda_j+i\omega}\frac{\chi_d^j(E)}{4\pi}\iint \nu_d^j(E')\frac{\delta\Sigma_f(\mathbf{r},E',\omega)}{\Sigma_f(\mathbf{r},E')}\Sigma_f(\mathbf{r},E')\psi_0(\mathbf{r},\mathbf{\Omega}',E')dE'\mathbf{\Omega}'
\end{aligned}
$$

General term:

$$
\underbrace{f_r(\mathbf{\Omega},E\to\mathbf{\Omega}',E')}_{\text{Emission spectrum}} \times \underbrace{\frac{\delta\Sigma_r(\mathbf{r},E,\omega)}{\Sigma_r(\mathbf{r},E)}}_{\text{Complex weight correction}} \times \underbrace{\Sigma_r(\mathbf{r},E)\psi_0(\mathbf{r},\mathbf{\Omega},E)}_{\text{Reaction rate}}
$$

Vibration of a finite-size region ($d > \Delta_L$):





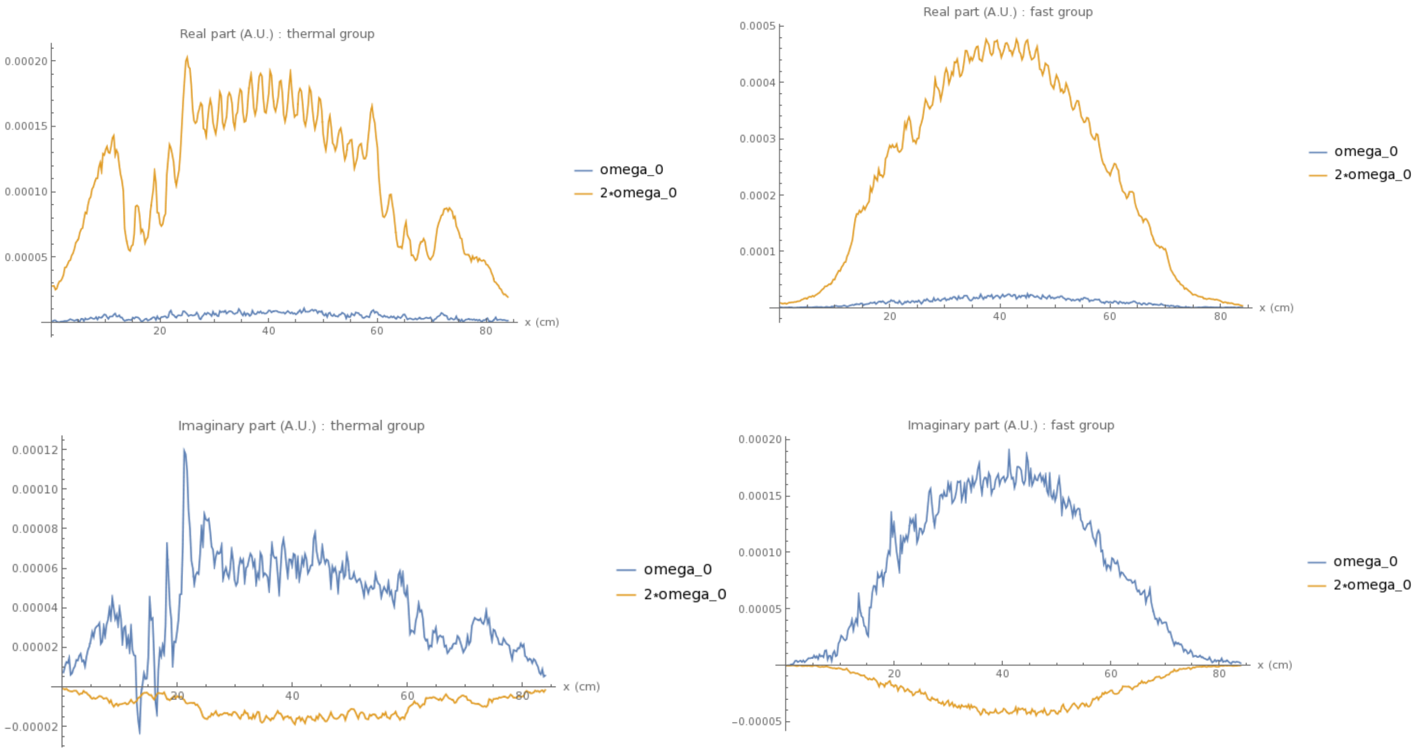
Amplitude:

For region 4: $$\frac{\delta\Sigma_4(t)}{\Sigma_4} = \frac{\Sigma_{\text{fuel}} - \Sigma_{\text{H}_2\text{O}}}{\Sigma_{\text{H}_2\text{O}}} H(r_0^{bd} + \varepsilon(t) - r)$$

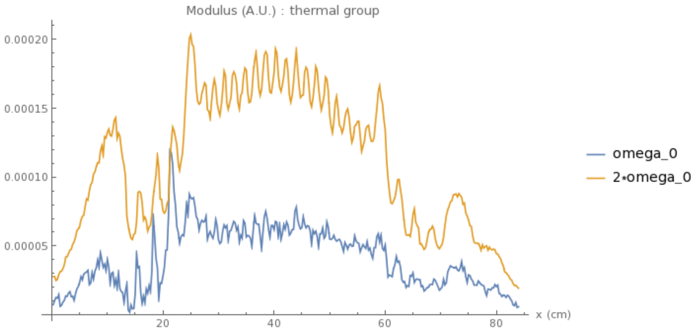
Fourier transform:
$$\delta\Sigma(r,\omega) = 2\Delta_\Sigma \left(\frac{\pi}{2} - \omega_0 \tau(r)\right) \delta(\omega) - 2\Delta_\Sigma \sum_{p=-\infty, p\neq 0}^{+\infty} \frac{\sin(2p\omega_0\tau(r))}{2p} \delta(\omega - 2p\omega_0) - 2i\Delta_\Sigma \sum_{p=-\infty}^{+\infty} \frac{\cos((2p+1)\omega_0\tau(r))}{2p+1} \delta(\omega - (2p+1)\omega_0)$$

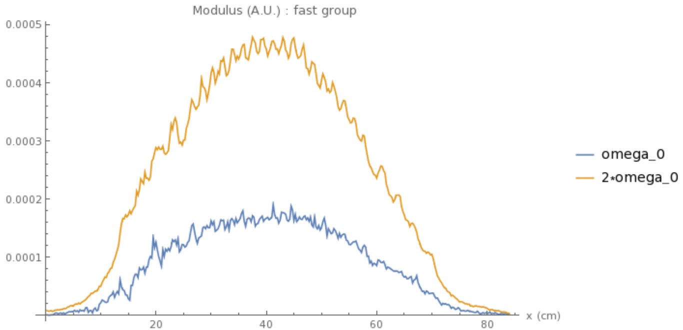
$$r = r_0^{bd} + \epsilon(\tau)$$

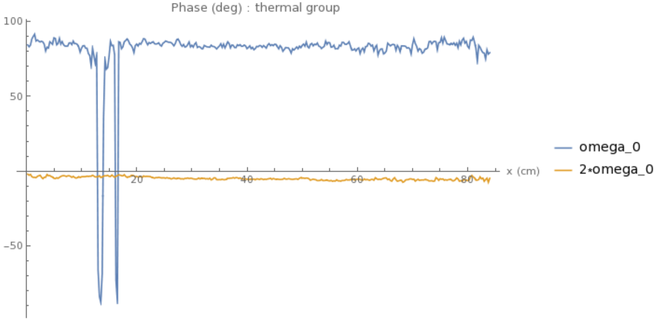
TRIPOLI-4 calculations: noise @ 1 Hz and 1.5 mm

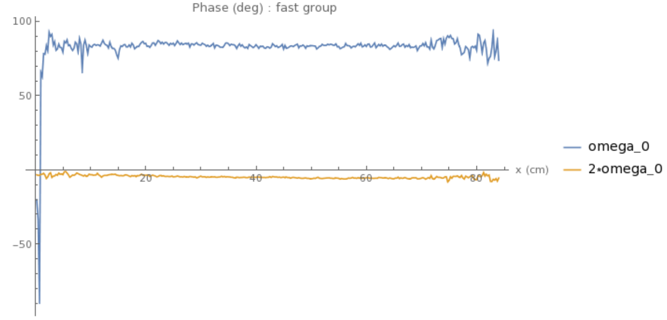


TRIPOLI-4 calculations: noise @ 1 Hz and 1.5 mm









HYPOTHESES

Assumption: the $2\omega_0$ noise component observed in all **experiments** is lower than the one measured at ω_0

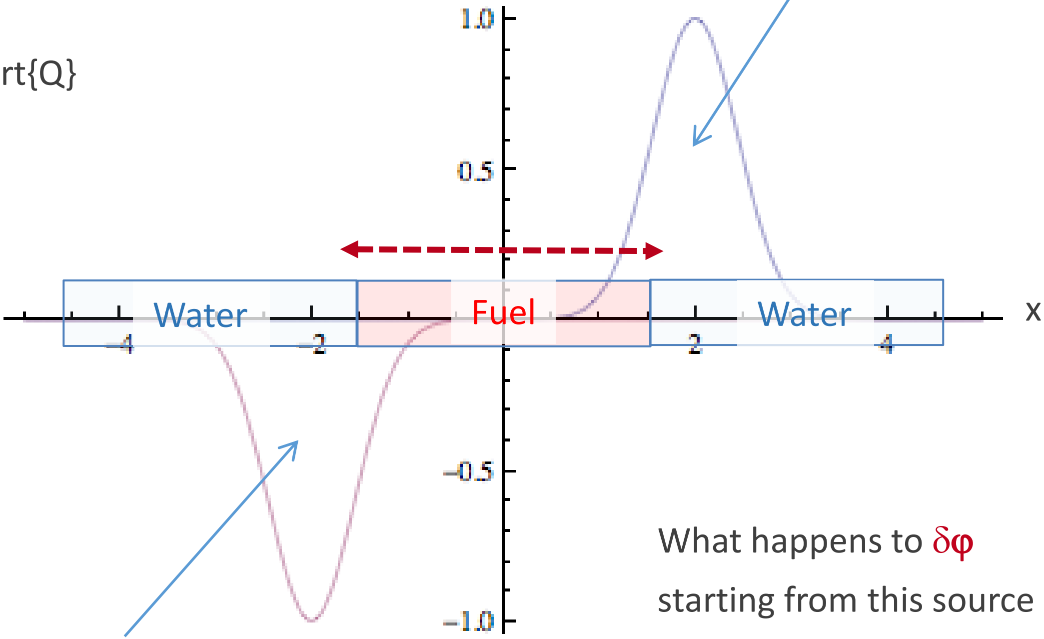
- Hp 1: there is a problem is the noise **solver**
  - However: it has been extensively verified
- Hp 2: there is a problem with the noise **source**
  - However: the derivation seems sound
- Hp 3: there is a problem with the **Fourier**-domain representation
  - Negative frequencies? Contribution at $\omega=0$?
- Hp 4: missing data in the **experiments**
  - Pendulum effect in the rods? Contribution of water vibration?

NOISE SOURCE Q(ω)

for a "typical" vibration

Real part{Q}

Positive density, **positive** weights



Positive density, **negative** weights

What happens to $\delta\varphi$ starting from this source ?

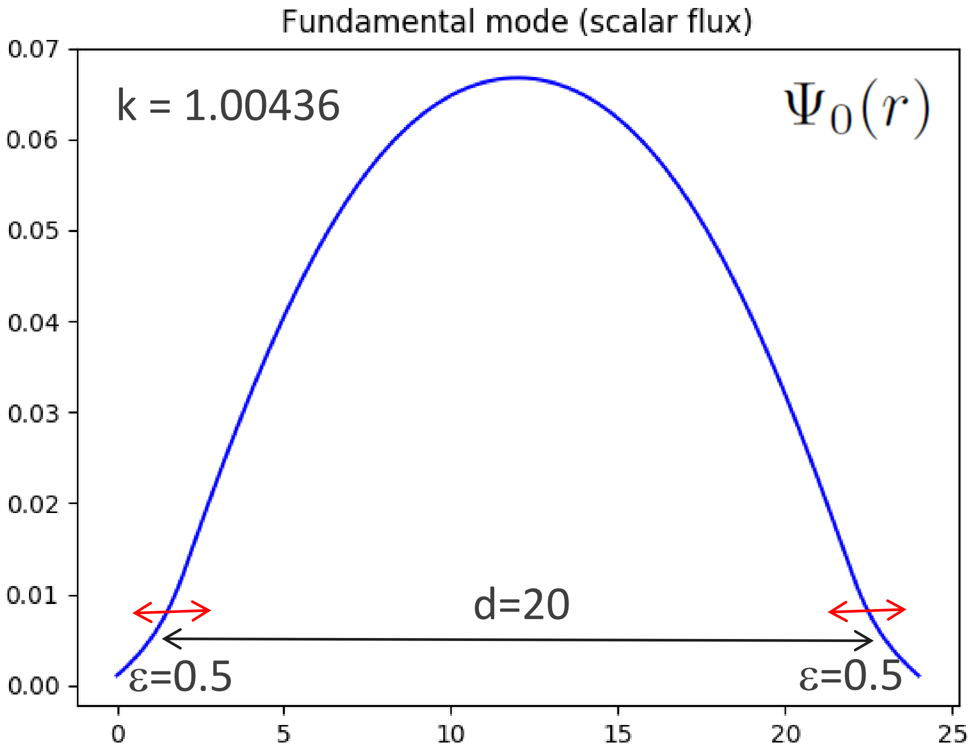
LARGE VIBRATING REGION: FUNDAMENTAL MODE

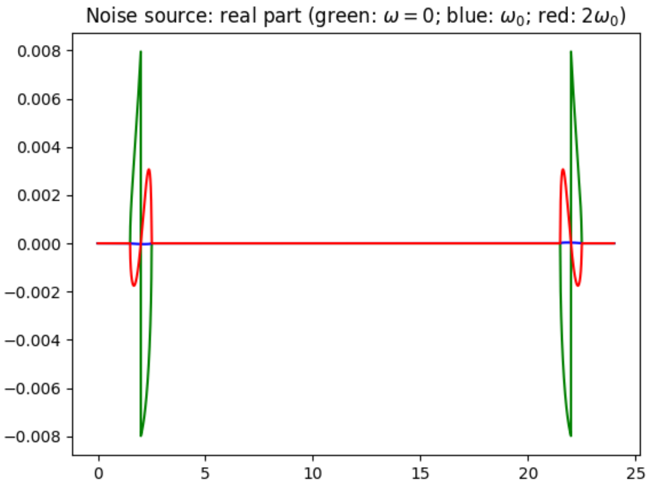
Domain = [0, 24]
x_L1 = 2.;
x_R1 = 22.;

sigs = {3., 1., 3.};
sigc = {0.2, 1.3, 0.2};
sigf = {0.0, 0.94, 0.0};

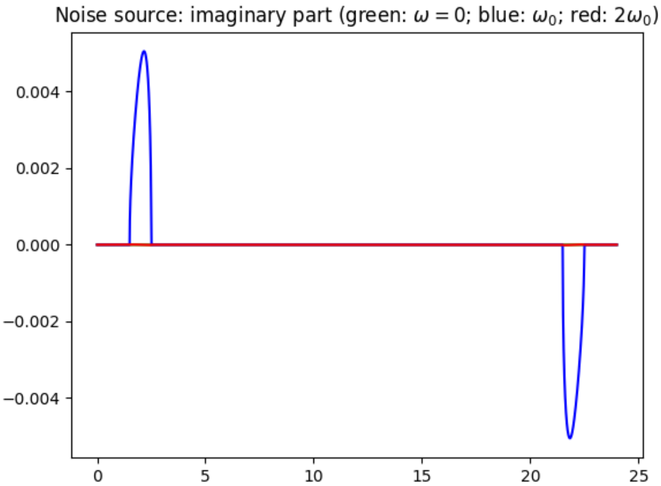
nu_f = 2.4;
beta = 7e-3;
lambda = 0.08;
speed = 2.2e5;

omega0 = 1.
eps = 0.5

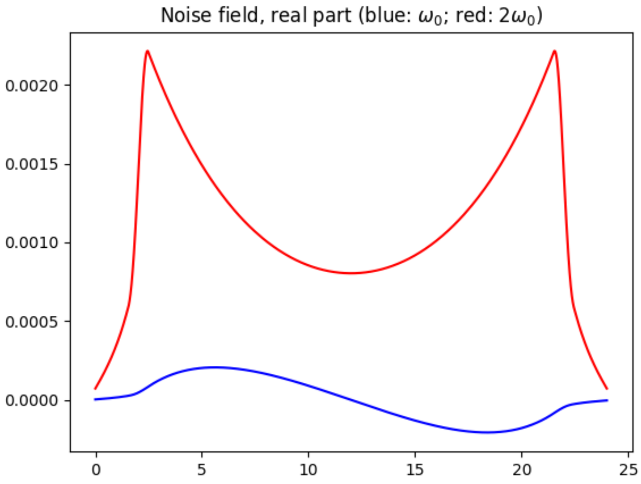




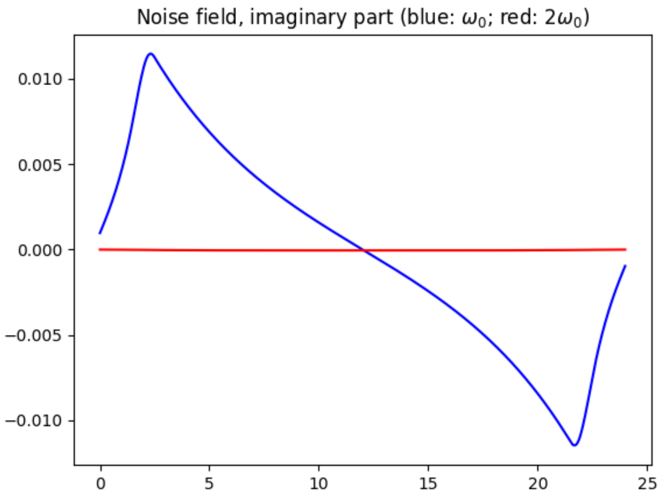
$$-\delta L(r,\omega)\Psi_0(r)$$



LARGE VIBRATING REGION: NOISE FIELD



$$\delta\Psi(r,\omega)$$



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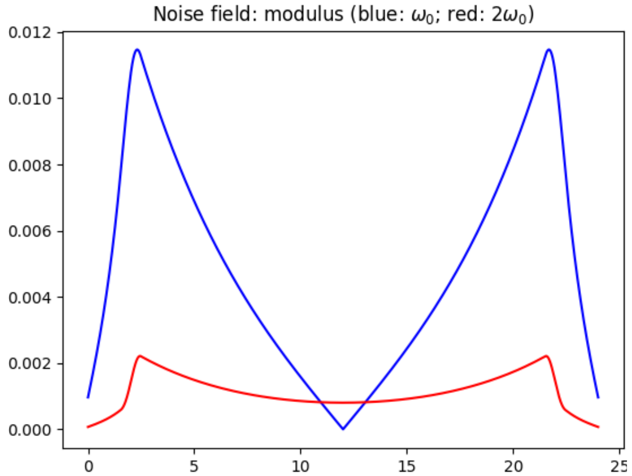
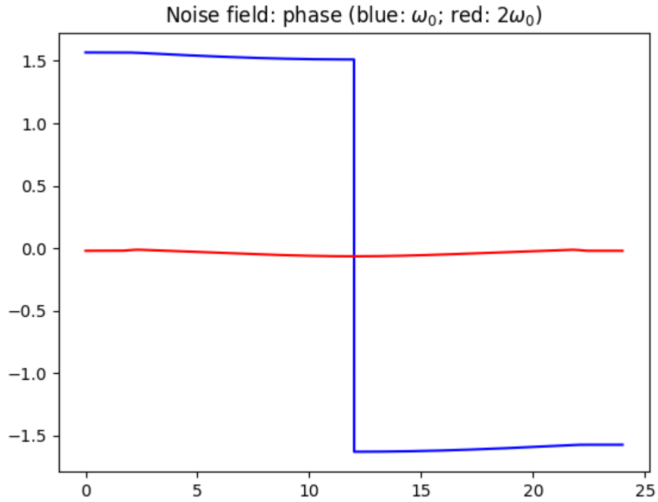


$$\delta\Psi(r, \omega)$$



Domain = [0, 10]
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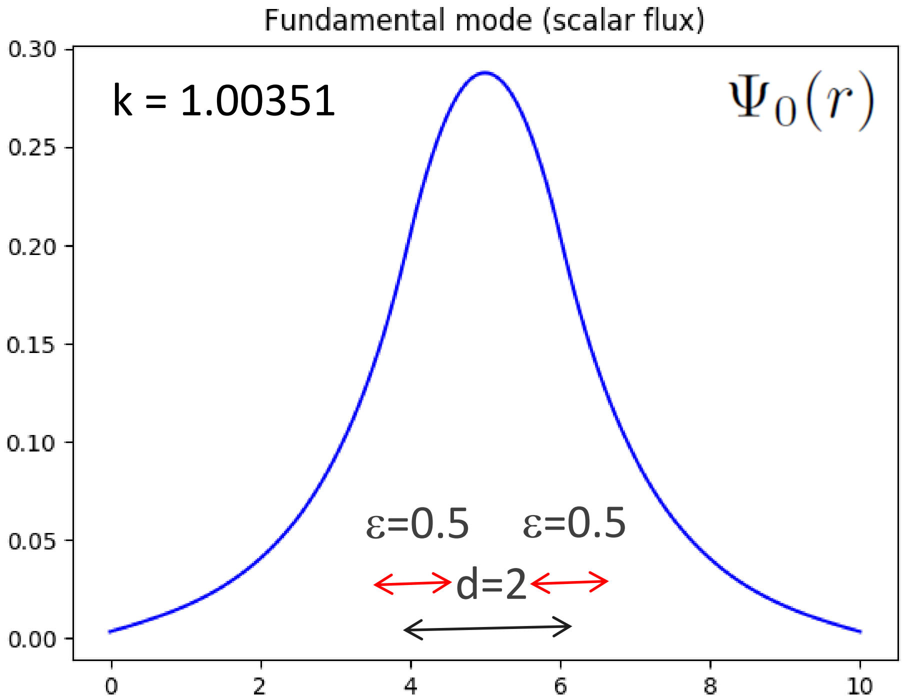
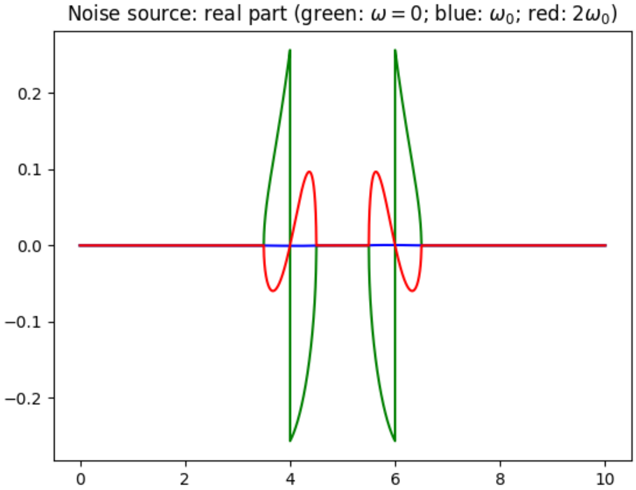
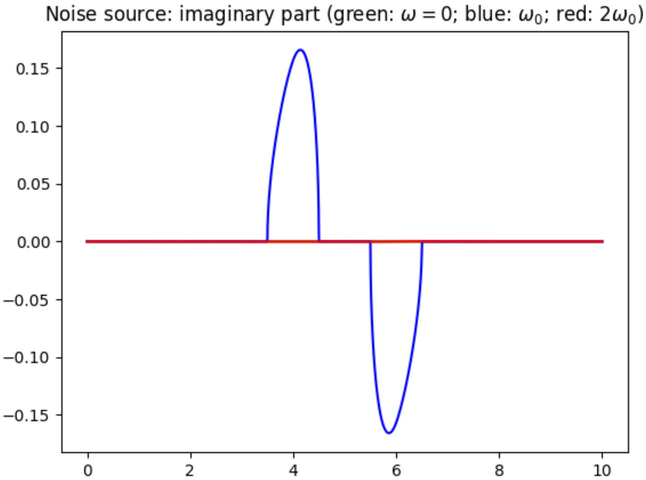
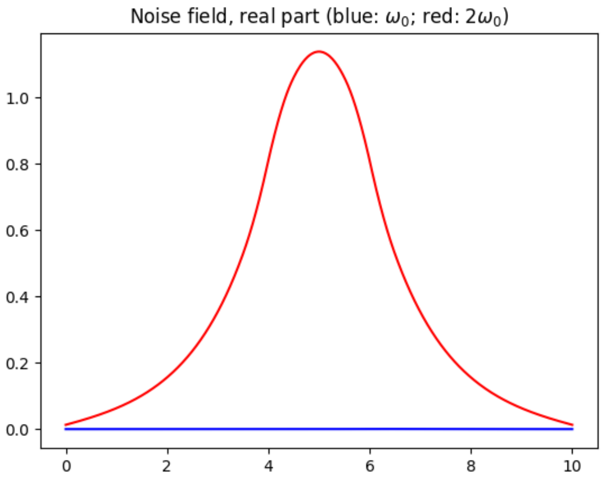


SMALL VIBRATING REGION: NOISE SOURCE

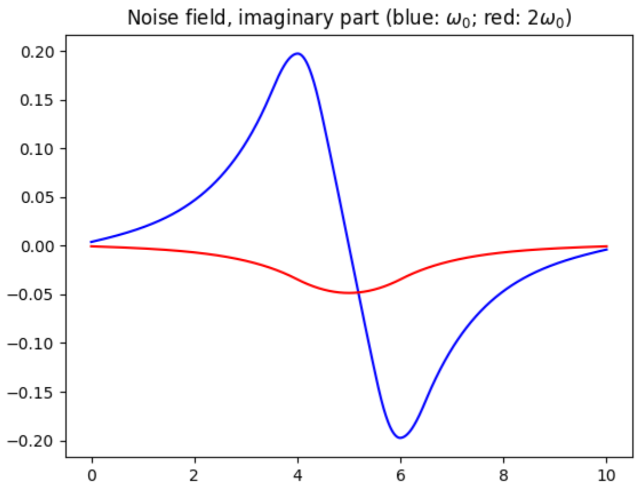


$$-\delta L(r,\omega)\Psi_0(r)$$





$$\delta\Psi(r,\omega)$$



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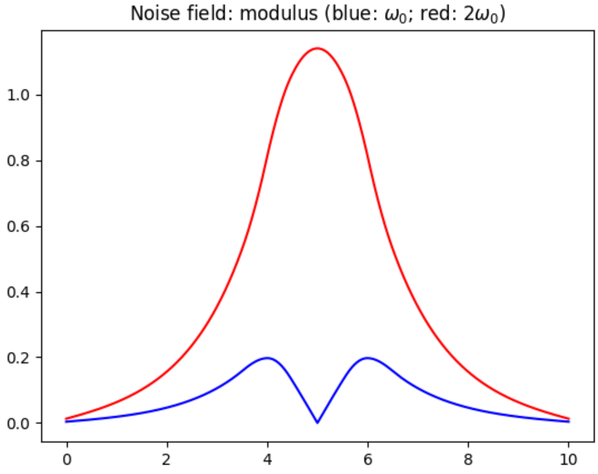
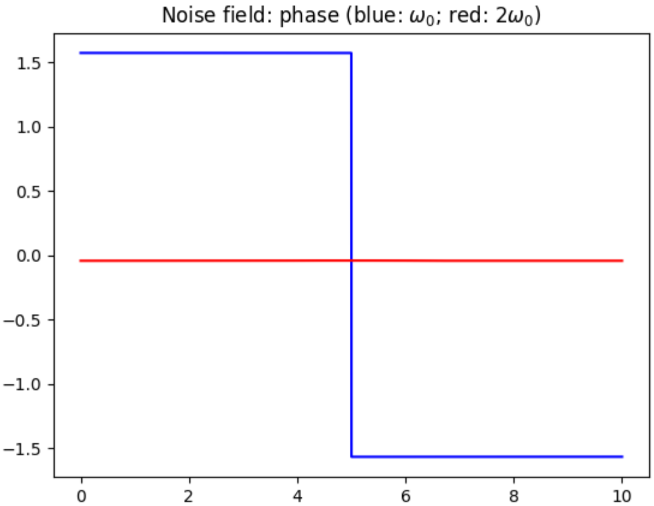
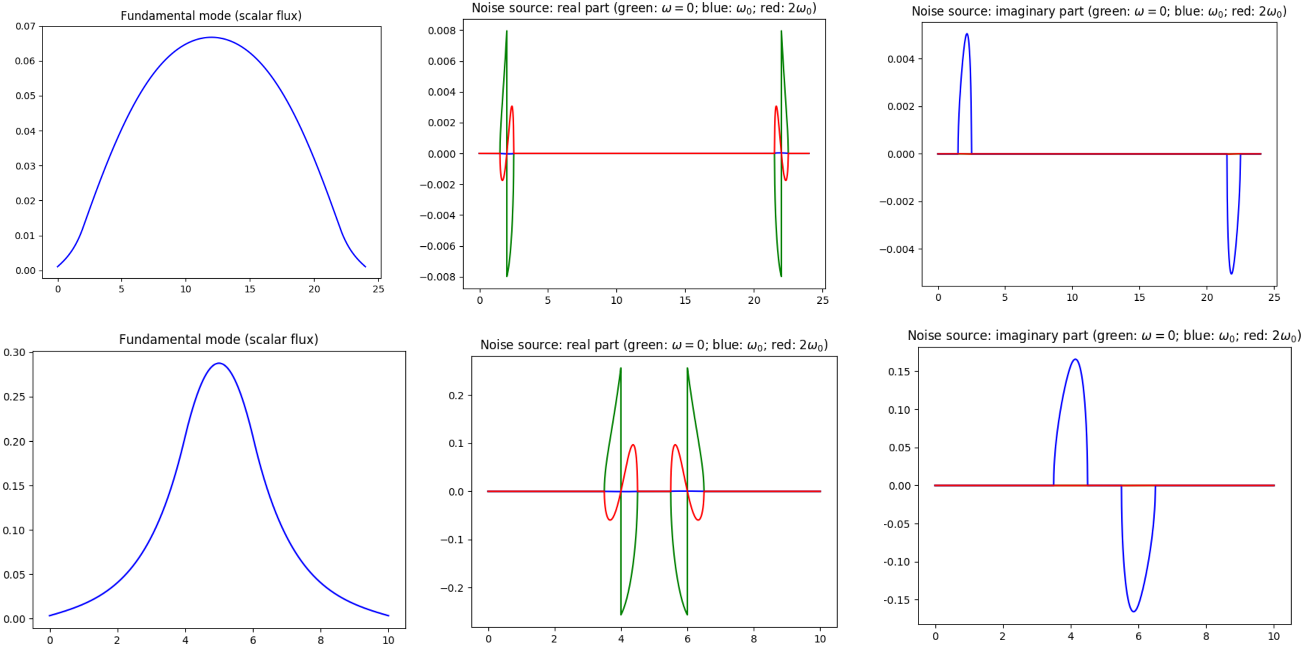


$$\delta\Psi(r,\omega)$$



Interplay of:

- Spatial shape, symmetries & separation of the noise source
- Spatial shape & gradients of the fundamental mode



REVISITING THE NOISE THEORY

Initial reactor state: $\left(\vec{\Omega}.\vec{\nabla} + \Sigma_0(r) - H_0(r) - P_0(r)\right)\Psi_0(r) = 0$

$$L_0 = \vec{\Omega}.\vec{\nabla} + \Sigma_0 - H_0 - P_0$$

Perturbation: $\left[\frac{1}{v}\partial_t + L(r,t)\right]\Psi(r,t) = 0$

Decomposition of the perturbed operator: $L(r,t) = L_0(r) + \delta L(r,t)$

Decomposition of the perturbed flux: $\Psi(r,t) = \Psi_0(r) + \delta\Psi(r,t)$

Exact noise equation: $\left[\frac{1}{v}\partial_t + L(r,t)\right]\delta\Psi(r,t) = -\delta L(r,t)\Psi_0(r)$

Hypothesis: neglecting the (small?) term $\delta L \delta\Psi$

Linearized noise equation $\left[\frac{1}{v}\partial_t + L_0(r)\right]\delta\Psi(r,t) = -\delta L(r,t)\Psi_0(r)$

Linearized noise equation in Fourier domain $L_{0,\omega}(r)\delta\Psi(r,\omega) = -\delta L(r,\omega)\Psi_0(r)$

AN IMPROVED (LINEAR) DECOMPOSITION

Introduce the **time-averaged** steady-state:

$$L_{0,\mathrm{NS}}(r) = \langle L \rangle (r) = L_0(r) + \langle \delta L \rangle (r)$$

Decomposition of the perturbed operator: $L_{\mathrm{NS}}(r,t) = L_{0,\mathrm{NS}}(r) + \delta L_{\mathrm{NS}}(r,t)$

Decomposition of the perturbed flux: $\delta L_{\mathrm{NS}}(r,t) = \delta L(r,t) - \langle \delta L \rangle (r)$

Linearized noise equation in Fourier domain:

$$L_{0,\mathrm{NS},\omega}(r) \delta \Psi_{\mathrm{NS}}(r,\omega) = -\delta L_{\mathrm{NS}}(r,\omega) \Psi_{0,\mathrm{NS}}(r)$$

Properties: $\langle \Psi_{\mathrm{NS}} \rangle (r) = \Psi_{0,\mathrm{NS}}(r)$, i.e., $\langle \delta \Psi_{\mathrm{NS}} \rangle (r) = 0$

- $\delta \Psi_{\mathrm{NS}}$ is the *smallest* periodic solution of the linearized noise equation

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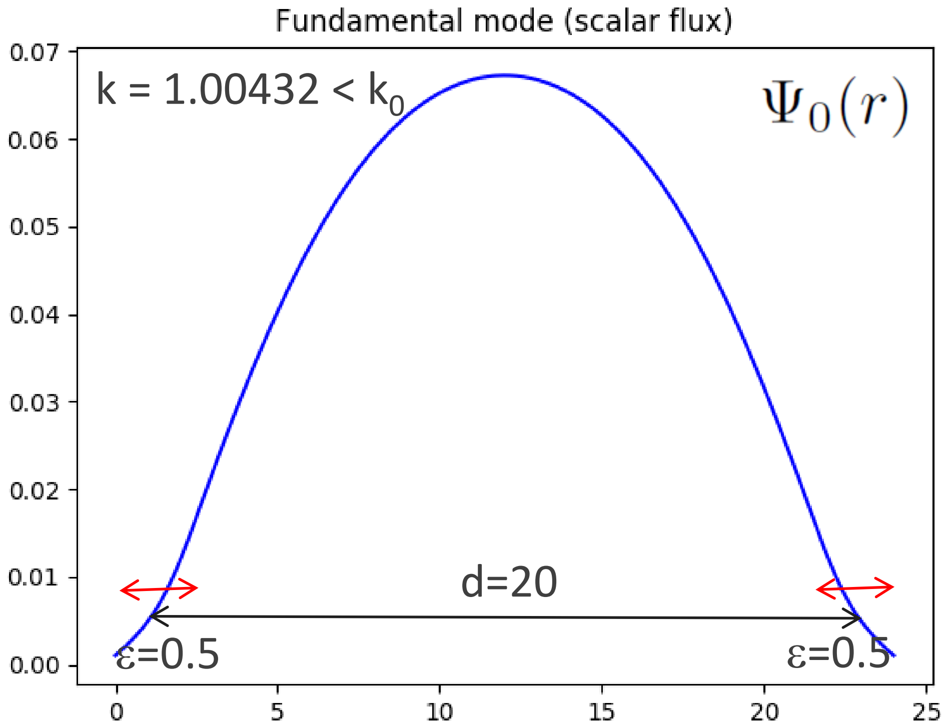
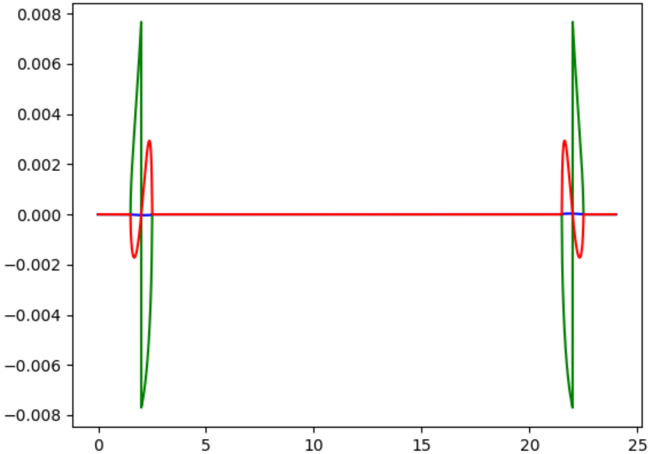
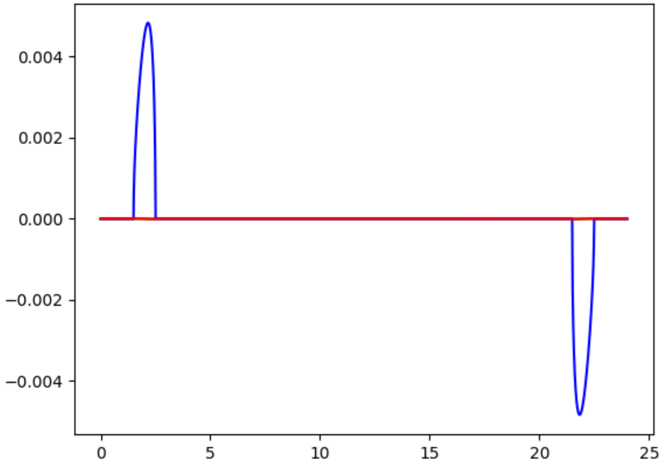


Noise source: real part (green: $\omega = 0$; blue: ω_0; red: $2\omega_0$)

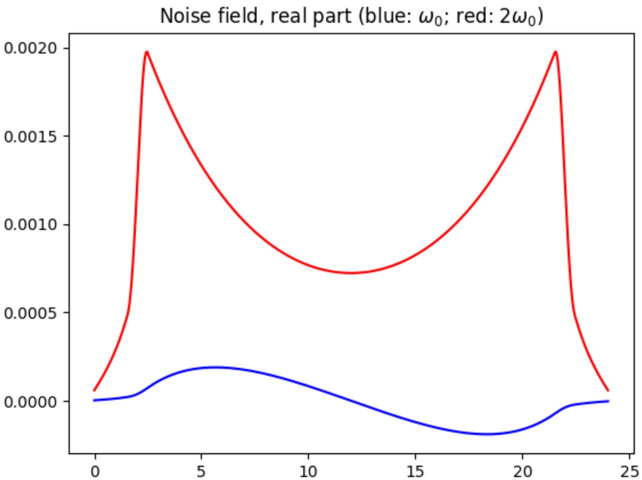


$$-\delta L(r, \omega)\Psi_0(r)$$

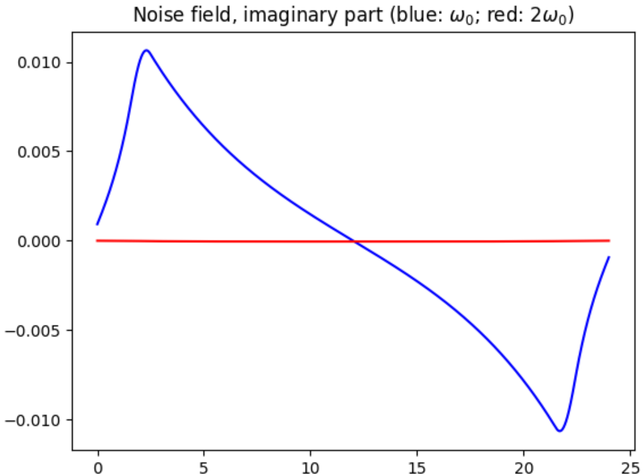
Noise source: imaginary part (green: $\omega = 0$; blue: ω_0; red: $2\omega_0$)



LARGE VIBRATING REGION: NOISE FIELD



$$\delta\Psi(r,\omega)$$



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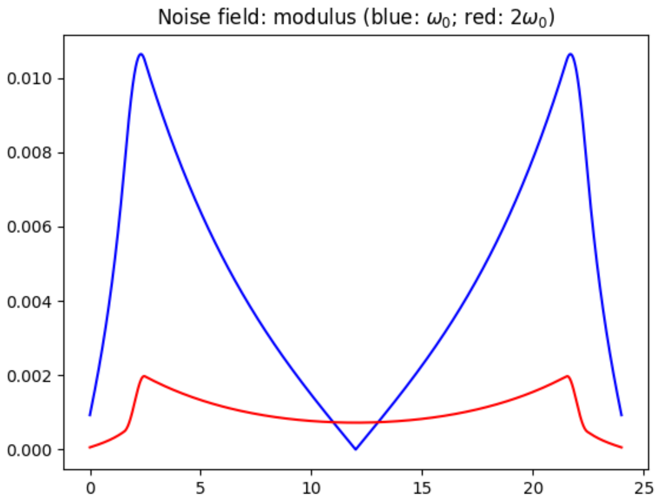

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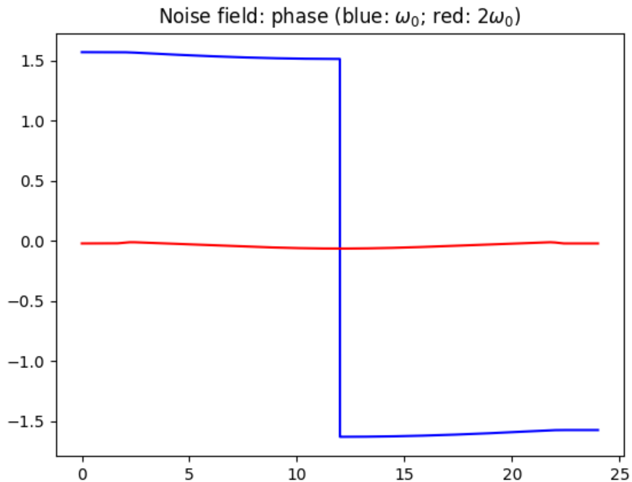

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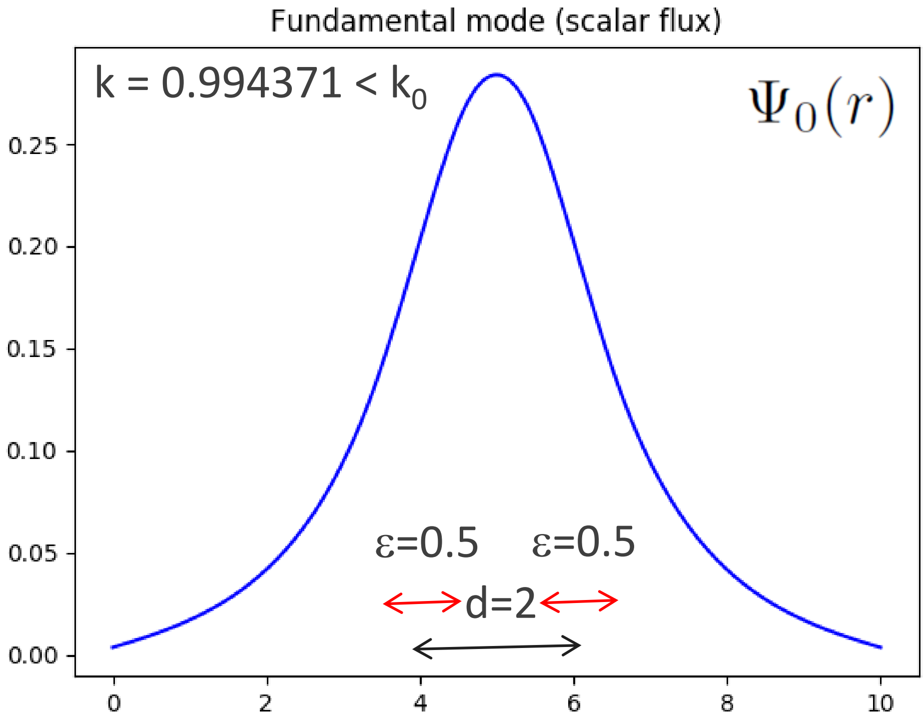
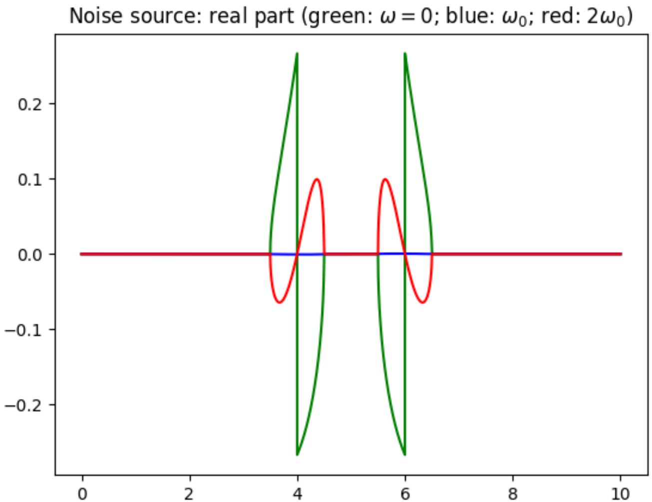
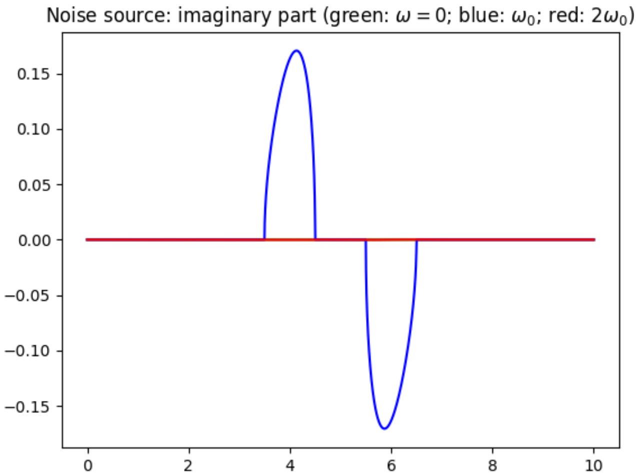


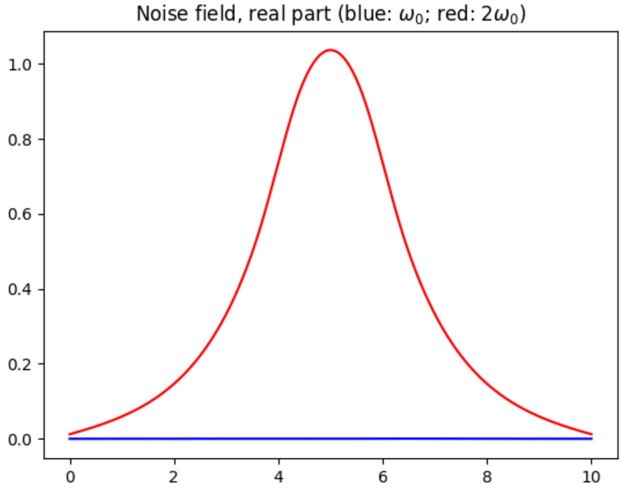
SMALL VIBRATING REGION: NOISE SOURCE



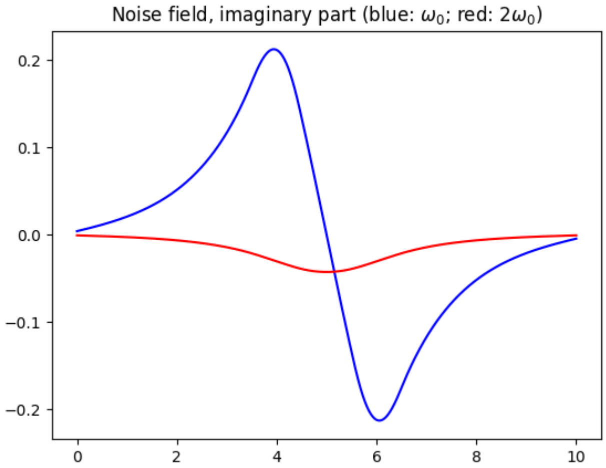
$$-\delta L(r,\omega)\Psi_0(r)$$



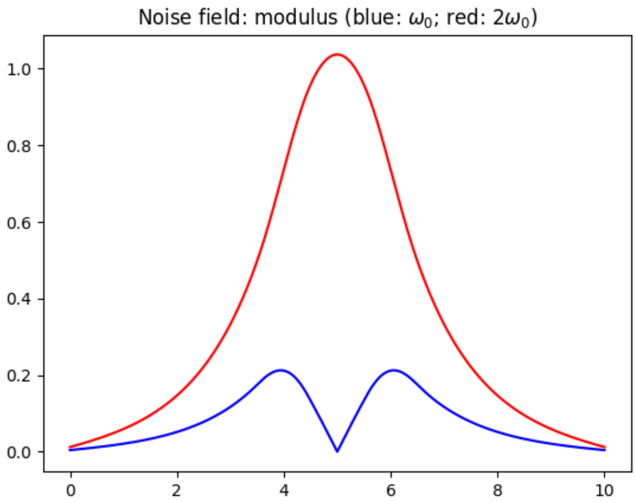
SMALL VIBRATING REGION: NOISE FIELD



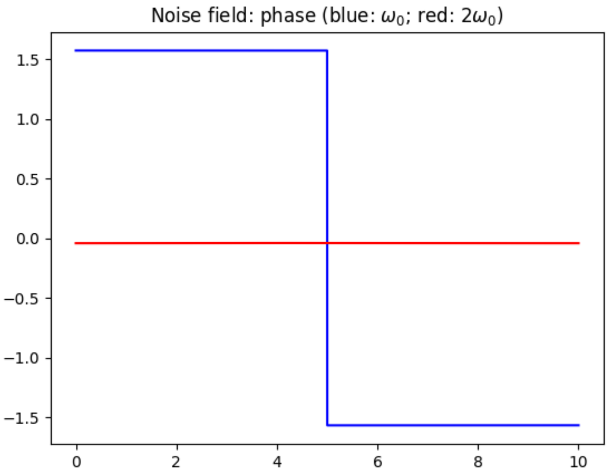
$$\delta\Psi(r,\omega)$$



SMALL VIBRATING REGION: NOISE FIELD



$$\delta\Psi(r,\omega)$$



Thank you for your attention