Power reactor noise

CORTEX joint WP1-WP2 Workshop
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Introduction

• Different communities use different “names”/“labels” in power reactor noise, leading to some confusion.
• In terms of modelling, many different approaches exist, all having advantages/disadvantages.
• Although the modelling of the induced neutron noise is essential, the modelling of the noise source is equally important.
Introduction

• Presentation thus focusing on:
  • Some theoretical remarks on power reactor noise
  • An overview of the various approaches for modelling the induced neutron noise
  • Noise source modelling
Some theoretical remarks on power reactor noise
Some theoretical remarks on power reactor noise

• Notations:
  all time-dependent terms split into a mean value and a fluctuating part as

$$ X (r,t) = X_0 (r) + dX (r,t) $$

with

$$ |dX (r,t)| = X_0 (r)," (r,t) $$

and

$$ \langle dX (r,t) \rangle = 0, " (r,t) $$
Some theoretical remarks on power reactor noise

• What is the point-kinetic component of the neutron noise?

Using the factorization:

\[ f(\mathbf{r}, t) = P(t) \times y(\mathbf{r}, t) \]

with

\[ P(t) \] amplitude factor
\[ y(\mathbf{r}, t) \] shape function

such that

\[ \frac{\partial}{\partial t} \dot{\phi}(\mathbf{r}) y(\mathbf{r}, t) d^3 \mathbf{r} = 0 \]
Some theoretical remarks on power reactor noise

- What is the point-kinetic component of the neutron noise?
  One obtains in first order:

  \[ df(r, t) = dP(t)f_0(r) + dy(r, t) \]

  where one assumed:

  \[ P_0 = 1 \]
  \[ y(r, t = 0) = f_0(r) \]

  - Point-kinetic response: \( dP(t)f_0(r) \)
  - “Space-dependent” response: \( dy(r, t) \)
Some theoretical remarks on power reactor noise

• What is the point-kinetic component of the neutron noise?

The fluctuations of the amplitude factor are further given, in the frequency domain, as:

\[
dP(w) = G_0(w)dr(w)
\]

with

\[
G_0(w) = \frac{1}{iwL_0 + \frac{b}{iw + l} \frac{\partial}{\partial t}}
\]

zero-power reactor transfer function

(better name: \textit{point-kinetic} zero-power reactor transfer function)
Some theoretical remarks on power reactor noise

• What is the point-kinetic component of the neutron noise?

Amplitude and phase of the zero-power reactor transfer function
Some theoretical remarks on power reactor noise

• What is the point-kinetic component of the neutron noise?

  Remark:
  Even at zero power (i.e. without feedback), the reactor response deviates from point-kinetics in the most general case
Some theoretical remarks on power reactor noise

• What are the local and global components of the reactor noise?

In more than or equal to two energy groups, the space-dependence of the induced neutron noise shows two relaxation lengths:
• A short relaxation length: the local component
• A long relaxation length: the global component
Some theoretical remarks on power reactor noise

• What are the local and global components of the reactor noise?

Example of the space-dependence of the amplitude of the thermal component of the Green’s function in two-group diffusion theory (at 5 Hz)
Some theoretical remarks on power reactor noise

• What are the local and global components of the reactor noise?

Remarks:
• The local component does not exist in one-group theory!
• The global component should not be mistaken with the point-kinetic component!
Some theoretical remarks on power reactor noise

• What are the local and global components of the reactor noise?

Illustration of the difference between the global component and the point-kinetic component (at 1 Hz)
Some theoretical remarks on power reactor noise

• Go to www.menti.com and use the code 60 79 61
Overview of the various approaches for modelling the induced neutron noise
Overview of the various approaches for modelling the induced neutron noise

• Once the noise source is modelled, need to estimate the response of the neutron flux to the applied perturbation

Could be done using the neutron transport equation (Boltzmann equation):

\[
\frac{1}{v(E)} \frac{\partial}{\partial t} y(r, W, E, t) = - W \times \mathbf{N}_y(r, W, E, t) - S_x(r, E, t) y(r, W, E, t)
\]

\[
+ \frac{\partial}{\partial t} \left( \mathbf{S}_x(r, W_{\mathbf{c}} \mathbf{W}, E, \mathbf{c}) E, t \right) y(r, W_{\mathbf{c}} E, \mathbf{c}) d^2 W \partial E \\mathbf{c}
\]

\[
+ \frac{1}{4p} \frac{\partial}{\partial \mathbf{c}} \left( \mathbf{c} \right) n(E, \mathbf{c}) S_f(r, E, \mathbf{c}) f(r, E, \mathbf{c}) \left( 1 - b \right) c^p(E) d(t - \mathbf{c}) + \sum_{i=1}^{N_d} \mathbf{a}_i \mathbf{L}_i \mathbf{b}_i e^{-1/(t - \mathbf{c})} d\mathbf{c} \partial E \\mathbf{c}
\]
Overview of the various approaches for modelling the induced neutron noise

• Neutron noise transport equation = integro-differential equations in the multi-dimensional phase space \((r, \mathbf{w}, E, t)\)

➢ Simpler formalisms usually used for modelling nuclear reactor cores, such as the multi-group diffusion approximation:

\[
\frac{1}{v_g} \frac{\partial f_g(r,t)}{\partial t} = \mathbf{N} \times \mathbf{D}_g(r,t) \mathbf{N}_f g(r,t) \mathbf{S}_{i,g}(r,t)f_g(r,t) \\
+ \sum_{g'=1}^{G} \mathbf{S}_{s0,g'\phi g'}(r,t)f_{g'}(r,t) + (1 - b)c_{g}^{p} \sum_{g'=1}^{G} n_{g'} S_{f,g'}(r,t)f_{g'}(r,t) + \sum_{i=1}^{N_d} l_{i,c_{i,g}} C_i(r,t)
\]

with

\[
\frac{\partial C_i(r,t)}{\partial t} = b_i \sum_{g'=1}^{G} n_{g'} S_{f,g'}(r,t)f_{g'}(r,t) - l_i C_i(r,t), i = 1, \ldots, N_d
\]
Overview of the various approaches for modelling the induced neutron noise

• Different approaches possible:
  • Time-domain modelling
    Advantages:
    ▪ Existing time-domain codes could be used
    ▪ Non-linear effects inherently accounted for
    ▪ Thermal-hydraulic feedback automatically taken into account

Disadvantages:
▪ Lengthy calculations
▪ Challenging to get a highly accurate solution for the noise
▪ Codes originally not developed for that purpose
▪ Lack of verification and validation for noise analyses
Overview of the various approaches for modelling the induced neutron noise

- Different approaches possible:
  - Frequency-domain modelling

  Time-domain equations transformed into frequency-domain equations according to the following procedure:

  - Splitting between mean values and fluctuations
  - Linear theory used because of the smallness of the fluctuations
  - Fourier-transform of the balance equations for the dynamical part only
Overview of the various approaches for modelling the induced neutron noise

• Different approaches possible:
  • Frequency-domain modelling
    Advantages:
    ▪ Codes specifically developed for noise analysis, thus usually fully verified (validated?)
    ▪ Highly accurate noise solution
    ▪ Usually high flexibility in the modelling
    ▪ Very fast calculations

Disadvantages:
▪ No commercial code available
▪ Possible linear effects disregarded
▪ Thermal-hydraulic feedback generally not taken into account (but could be)
Overview of the various approaches for modelling the induced neutron noise

- Codes used in CORTEX:

<table>
<thead>
<tr>
<th>Code name</th>
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<th>Spatial resolution</th>
<th>Approach</th>
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<tbody>
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<td>Time</td>
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Overview of the various approaches for modelling the induced neutron noise

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Noise source modelling
Noise source modelling

• Since all neutron transport codes use nuclear macroscopic cross-sections as input, need to convert “physical” perturbations into perturbations of macroscopic cross-sections.

• Perturbations can be defined:
  • In the time-domain, more or less as they are, with limitations/approximations due to the mesh used.
  • In the frequency-domain, after typically a first-order approximation of the perturbation, subsequently followed by a Fourier transform + limitations/approximations due to the mesh used.

➢ Modelling possibly supplemented by other modelling tools (e.g. fluid-structure modelling tool)

➢ Noise source modelling strongly dependent on the choices made by the user
Noise source modelling

• Different scenarios investigated in CORTEX:
  • “Absorber of variable strength”
  • “Vibrating absorber”
  • Axially-travelling perturbations
  • Inlet flow rate perturbations
  • Fuel assembly vibrations
  • Core barrel vibrations
Noise source modelling

• “Absorber of variable strength” type of noise source:
  • Localized perturbation of which its amplitude varies in time at a fixed position
  • Induced neutron noise given by the Green’s function
  • The effect of all other types of noise source can be given using such Green’s functions
  • Simplest type of perturbation but “academic” type of perturbation

NB: An inlet flow rate perturbation can be seen as an absorber of variable strength type of perturbation distributed along the height of the perturbed channel.
Noise source modelling

• “Vibrating absorber” type of noise source:
  • Lateral movement of the absorber represented as (weak absorber):
    \[
    dS_{a,2}(\mathbf{r},t) = gq(z - z_0)\hat{g}(\mathbf{r}_{xy} - \mathbf{r}_{p,xy} - \mathbf{e}(t)) - d(\mathbf{r}_{xy} - \mathbf{r}_{p,xy})^2
    \]
  • A first-order Taylor expansion of the noise source would result in the induced neutron noise (in the frequency-domain) given by the gradient of the Green’s function with respect to the equilibrium position \( \mathbf{r}_{p,xy} \) of the moving rod.
Noise source modelling

• Axially-travelling noise source at a velocity $v$:
  • Noise sources given as “absorber of variable strength” type of noise sources, spatially distributed along the perturbed channel, and time shifted at the location $z$ by $(z - z_0)/v$, with $z_0$ axial location of the inlet
**Noise source modelling**

- Fuel assembly vibrations:
  - Different possible axial vibration modes for fuel assemblies:

<table>
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<tr>
<th>Axial shape of the displacement $d(z,t)$ in arbitrary units as a function of the relative core elevation $z$</th>
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Noise source modelling

- Fuel assembly vibrations:
  - Can be modelled at the pin level: either as “vibrating absorbers” or as “absorbers of variable strength”.
  - Can only be modelled at the nodal level as “absorber of variable strength”.

Noise source modelling

• Fuel assembly vibrations:
  • Lateral vibrations represented as:

• The modelling requires a model of the displacements along the $x$ and $y$ directions.
Noise source modelling

• Fuel assembly vibrations:
  • A first-order approximation leads to the modelling of the vibrations along a given direction as two noise sources being out-of-phase and located at the boundary between the vibrating fuel assembly and its two (non-moving) neighbours.
  • A refinement of the mesh to describe the displacement of a structure induces the appearance of higher harmonics.
Noise source modelling

• Core barrel vibrations:
  • Can be seen as a relative displacement of the active core with respect to the reflector:

➢ Modelling identical to the case of vibrating fuel assemblies.
Conclusions and outlook

• Modelling the effect of noise sources can be done in many ways:
  • Time-domain/frequency-domain
  • Diffusion/transport
  • Deterministic/probabilistic
  • Fine/coarse spatial mesh

• Taking full advantage of noise analysis requires:
  • A correct modelling of the noise source
  • The estimation of the reactor transfer function
  • Its inversion
Power reactor noise

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Backup slides
“Absorber of variable strength” type of noise source

• “Absorber of variable strength” = localized perturbation of which its amplitude varies in time at a fixed position

• Induced neutron noise given by the following balance equation (2-group diffusion theory):

$$\left\{ \tilde{N} \times \tilde{P}(r) \tilde{N} \hat{\eta}^T + S_{dyn}(r, w) \right\} \hat{u}^{eff} = \hat{u}^{eff 1} (r, w) + \hat{u}^{eff 2} (r, w)$$

$$= f_r (r) dS_r (r, w) + f_a (r) \hat{S}_{a,1} (r, w) + f_f (r, w) \hat{S}_{f,1} (r, w) + f_f (r, w) \hat{S}_{f,2} (r, w)$$
“Absorber of variable strength” type of noise source

• In case of a point-like source:

\[
\hat{\mathbf{N}}_r \times \mathcal{D}(\mathbf{r}) \hat{\mathbf{N}}_r \hat{\mathbf{u}} + S_{\text{dyn}}(\mathbf{r}, \mathbf{w}) \hat{\mathbf{u}} = \begin{cases} \hat{z}(\mathbf{r} - \mathbf{r}_0) \hat{\mathbf{u}} & \text{or} \quad \begin{cases} 0 \hat{\mathbf{u}} & q = 1 \\ \hat{z}(\mathbf{r} - \mathbf{r}_0) \hat{\mathbf{u}} & q = 2 \end{cases} \
\end{cases}
\]

➢ Green’s function
“Absorber of variable strength”

Type of noise source

General solution to the original problem can be given by convolution integrals

\[
\hat{g}_1(f, w) = \int G_1(r, r \cdot w) S_1(r \cdot w) + G_2(r, r \cdot w) S_2(r \cdot w) \, d^3 r \cdot w
\]

\[
\hat{g}_2(f, w) = \int G_1(r, r \cdot w) S_1(r \cdot w) + G_2(r, r \cdot w) S_2(r \cdot w) \, d^3 r \cdot w
\]

with

\[
\hat{f}_1(r \cdot w) = f_r(r \cdot w) dS_r(r \cdot w) + f_a(r \cdot w) dS_{a,1}(r \cdot w) + f_f(r \cdot w) dS_{f,1}(r \cdot w)
\]

\[
\hat{f}_2(r \cdot w) = f_r(r \cdot w) dS_r(r \cdot w) + f_a(r \cdot w) dS_{a,2}(r \cdot w) + f_f(r \cdot w) dS_{f,2}(r \cdot w)
\]
“Absorber of variable strength” type of noise source

• Example of a localized “absorber of variable strength” @ 1kHz
“Vibrating absorber” type of noise source

- Lateral movement of the absorber represented as (weak absorber):
  \[ dS_{a,2}(r,t) = gq(z - z_0)\hat{g}(r_{xy} - r_{p,xy} - e(t)) - d(r_{xy} - r_{p,xy}) \]

- A first-order Taylor expansion of the noise source would give for the induced neutron noise (in the frequency-domain):
  \[ df_g(r, w) = - ge(w) \times dj_g(r, w) \]

  with
  \[ dj_g(r, w) = \tilde{N}_{r, p,xy} \hat{G}_{2}^{g}(r, r_{p,xy}, w) \]
“Vibrating absorber” type of noise source

• Example of a vibrating control rod @ 0.2 Hz

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Axially-travelling perturbations

- Noise source represented in the time-domain as:

\[
\begin{align*}
    dS_{\text{rem}} (r,t) &= dS_{\text{rem}} (x,y,z,t) \\
    &= \begin{cases} 
    0, & \text{if } (x,y) = (x_0,y_0) \text{ and } z < z_0 \\
    0, & \text{if } (x,y) = (x_0,y_0) \text{ and } z < z_0 \\
    \int_{\partial V_{x_0,y_0,z_0,t}} dS_{\text{rem}} \left( x_0, y_0, z_0, t \right) - \frac{z - z_0}{v} \frac{\partial}{\partial t} & \text{if } (x,y) = (x_0,y_0) \text{ and } z \geq z_0
    \end{cases}
\end{align*}
\]
Axially-travelling perturbations

• Noise source represented in the frequency-domain as:

\[
\begin{align*}
\frac{dS_{rem}(r,w)}{dS_{rem}(x,y,z,w)} &= \\
&= \begin{cases} 
0, & \text{if } (x,y) = (x_0,y_0) \\
0, & \text{if } (x,y) = (x_0,y_0) \text{ and } z < z_0 \\
\left[dS_{rem}(x_0,y_0,z_0,w) \exp\left(iw(z - z_0)\frac{u}{v}\right)\right], & \text{if } (x,y) = (x_0,y_0) \text{ and } z > z_0 \end{cases}
\end{align*}
\]
Axially-travelling perturbations

• Example of a travelling perturbation @ 1Hz

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Fuel assembly vibrations

- Different possible axial vibration modes for fuel assemblies:

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Fuel assembly vibrations

• Fuel assembly vibrations described at the pin level:
  – Can be modelled as “vibrating absorbers”
  – Can be modelled as “absorbers of variable strength”!

• Fuel assembly vibrations at the nodal level can only be modelled as “absorber of variable strength”!
Fuel assembly vibrations

• Lateral vibrations represented as:
Fuel assembly vibrations

• In e.g. the $x$ -direction, one has:

with static cross-section between Regions II and III given as:

$$S_{a,g}^{x} (x) = \frac{Q}{g} - Q (x - b)S_{a,g,II} + Q (x - b)S_{a,g,III}$$
Fuel assembly vibrations

• For a time-dependent boundary:

\[ b(z, t) = b_0 + e_x(z, t) \]

one obtains after a first-order Taylor expansion in the time-domain:

\[
S^x_{a,g}(x, z, t) = Q(x - b_0) \hat{S}^g_{a,g,II} + Q(x - b_0) S^g_{a,g,III} + e_x(z, t) d(x - b_0) \hat{S}^g_{a,g,II} - S^g_{a,g,III} \hat{u} e_x(z, t)
\]

➢ Noise source in the frequency-domain:

\[
dS^x_{a,g}(x, z, w) = e_x(z, w) d(x - b_0) \hat{S}^g_{a,g,II} - S^g_{a,g,III} \hat{u}
\]

➢ Point-like source!
Pendular core barrel vibrations

- Core barrel vibrations can be seen as a relative displacement of the active core with respect to the reflector:
Pendular core barrel vibrations

➢ Same technique as for fuel assembly vibrations can be used:

\[ dS_{a,g}^x (x, z) = h(z) \hat{a}_n d(x - x_n) \hat{S}_{a,g,x_n} - S_{a,g,x_n} \hat{u} \]

\[ dS_{a,g}^y (y, z) = h(z) \hat{a}_n d(y - y_m) \hat{S}_{a,g,y_m} - S_{a,g,y_m} \hat{u} \]

➢ Point-like source!
Estimation of the induced neutron noise

• Generically, the induced neutron noise is given as (e.g. in 2-group theory):

\[
\begin{align*}
\dot{G}_{1}^{\text{eff}} (r, w) &= \dot{G}_{1}^{\text{G}} (r, r \mathcal{G} w) S_{1} (r \mathcal{G} w) + G_{2}^{\text{G}} (r, r \mathcal{G} w) S_{2} (r \mathcal{G} w) d^{3} \mathcal{G} w \\
\dot{G}_{2}^{\text{eff}} (r, w) &= \dot{G}_{2}^{\text{G}} (r, r \mathcal{G} w) S_{1} (r \mathcal{G} w) + G_{2}^{\text{G}} (r, r \mathcal{G} w) S_{2} (r \mathcal{G} w) d^{3} \mathcal{G} w \\
\end{align*}
\]

with

\[
\begin{align*}
\dot{G}_{1}^{s} (r \mathcal{G} w) &= f_{r} (r \mathcal{G}) dS_{r} (r \mathcal{G} w) + f_{a} (r \mathcal{G}) \dot{G}_{a,1}^{s} (r \mathcal{G} w) \\
\dot{G}_{2}^{s} (r \mathcal{G} w) &= f_{a} (r \mathcal{G}) \dot{G}_{a,2}^{s} (r \mathcal{G} w) + f_{f} (r \mathcal{G} w) \dot{G}_{f,1}^{s} (r \mathcal{G} w) \\
\end{align*}
\]
Estimation of the induced neutron noise

... or given as:

\[
df_g (r, w) = - g e(w) \times dj_g (r, w)
\]

with

\[
dj_g (r, w) = \tilde{N}_{r, p, xy} \hat{G}_{2\&g} (r, r_{p, xy}, w)
\]

➢ In essence, only the Green’s function is needed
Estimation of the induced neutron noise

• The Green’s function can be estimated:
  • Either deterministically
    • Using diffusion theory
      \[
      \hat{G}_{\text{r}} (r, \text{r} \, \hat{g}) \hat{N}_r + S_{\text{dyn}} (r, w) \hat{u} = \hat{G}_{\text{r}} (r, \text{r} \, \hat{g}) \hat{u} = \hat{G}_{\text{r}} (r, \text{r} \, \hat{g}) \hat{d} (r - \text{r} \, \hat{g}) \hat{u} \quad \text{or} \quad \hat{G}_{\text{r}} (r, \text{r} \, \hat{g}) \hat{d} (r - \text{r} \, \hat{g}) \hat{u} = 0 \quad \hat{u} = 1 \quad \text{or} \quad \hat{G}_{\text{r}} (r, \text{r} \, \hat{g}) \hat{d} (r - \text{r} \, \hat{g}) \hat{u} = 0 \quad \hat{u} = 2
      \]
  • Using transport theory
      \[
      \hat{G}_{\text{r}} (r, \text{r} \, \hat{g} \, w) \hat{N}_r + S_{\text{dyn}} (r, \text{r} \, \hat{g} \, w) \hat{u} = \hat{G}_{\text{r}} (r, \text{r} \, \hat{g} \, w) \hat{u} = \hat{G}_{\text{r}} (r, \text{r} \, \hat{g} \, w) \hat{d} (r - \text{r} \, \hat{g} \, w) \hat{u} \quad \text{or} \quad \hat{G}_{\text{r}} (r, \text{r} \, \hat{g} \, w) \hat{d} (r - \text{r} \, \hat{g} \, w) \hat{u} = 0 \quad \hat{u} = 1 \quad \text{or} \quad \hat{G}_{\text{r}} (r, \text{r} \, \hat{g} \, w) \hat{d} (r - \text{r} \, \hat{g} \, w) \hat{u} = 0 \quad \hat{u} = 2
      \]
Estimation of the induced neutron noise

- The Green’s function can also be estimated:
  - Probabilistically:
    - Introducing complex weights in the Monte Carlo solver
    - Requires modification of the source codes
Estimation of the induced neutron noise

- The Green’s function can also be estimated:
  - Probabilistically:
    - Using an equivalence to subcritical problems (demonstrated hereafter on diffusion theory in 2 energy groups):
      \[
      \hat{\chi}_{\text{eff} 1} (r, w) \hat{u} = \hat{\chi}_{\text{eff} \text{ real} 1} (r, w) \hat{u} + i \hat{\chi}_{\text{eff} \text{ im} 1} (r, w) \hat{u}
      \]
      \[
      \hat{\chi}_{\text{eff} 2} (r, w) \hat{u} = \hat{\chi}_{\text{eff} \text{ real} 2} (r, w) \hat{u} + i \hat{\chi}_{\text{eff} \text{ im} 2} (r, w) \hat{u}
      \]
      The coupling to the imaginary (real, respectively) part of the neutron noise when solving for the real (imaginary, respectively) balance equations is treated as additional noise sources
      \[
      \hat{S}_{\text{real or im} 1} (r, w) \hat{u} = \text{Re or Im} \left\{ \hat{S}_{1} (r, w) \hat{u} \right\} + \text{Im or Re} \left\{ \hat{df}_{1} (r, w) \hat{u} \right\}
      \]
      \[
      \hat{S}_{\text{real or im} 2} (r, w) \hat{u} = \text{Re or Im} \left\{ \hat{S}_{2} (r, w) \hat{u} \right\} + \text{Im or Re} \left\{ \hat{df}_{2} (r, w) \hat{u} \right\}
      \]
    - No modification of the source code required.