

ASSESSMENT OF THE NEUTRON NOISE INDUCED BY STATIONARY FUEL ASSEMBLY VIBRATIONS IN A LIGHT WATER REACTOR

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ABSTRACT

A systematic increase of the neutron noise levels over time has been observed in some of the pre-KONVOI PWRs operating in Europe. A possible reason for this anomaly was identified as increased mechanical vibrations of reactor internals, specifically of fuel assemblies. To verify this conjecture, the modeling of stationary vibrations of fuel assemblies and of the corresponding neutron noise is essential. In this paper, using the ϵ/d model basis, we illustrate the modeling of the neutron noise sources for fuel assembly vibrations and study the effect of homogenization of cross sections on such stationary perturbations. A comparative analysis between the classical nodal approach (both localized at the boundaries of the vibrating fuel assembly or involving the entire neighboring fuel assemblies) and a pin-wise approach shows that the ‘boundary-localized’ nodal approach seems to capture local noise information, as a pin-wise approach would do, without the need of a complex pin-by-pin model when the detectors are placed close to the perturbation. However, when considering region of the active core away from the perturbation, all three approaches lead to comparable results.

KEYWORDS: Neutron noise, noise analysis, fuel assembly vibration, frequency domain, CORE SIM

1. INTRODUCTION

The analysis of stationary perturbations in nuclear reactors and of their effect on the neutron flux is often subsumed under the term of noise analysis, with noise formally referring to the deviation of any time-dependent variable from its mean value. The modeling of stationary perturbations and of

the corresponding neutron noise has received renewed interest from the reactor physics community in recent years, as a systematic increase of the neutron noise levels with time has been observed in some of the pre-KONVOI PWRs operating in Europe [1]. In those plants, the increase of the noise level was such that it had an impact on the availability of the plants. Although the reasons for the operational problems remain partially unexplained [2], increased mechanical vibrations of reactor internals and more specifically of fuel assemblies were identified as one of the possible root causes of the increased noise levels over several cycles.

The effect of vibration of fuel assemblies impacts local as well as global parameters such as reactivity and total neutron flux. With a prior knowledge of the sensitivity of such perturbation initiators, realistic alert levels can be determined, from which justification can be derived for continued reactor operation. Therefore, from an industrial perspective, the possibility of predicting, identifying and locating fuel assemblies vibrations is of keen interest in order to ensure optimum operation of the plant. The CORTEX project (CORE monitoring Techniques and EXperimental validation and demonstration), which is funded by the European Commission, aims to develop a core monitoring technique that is able to retrieve the location of anomalies and their characteristic features from in-core and ex-core instrumentation [3]. The CORTEX project also intends to identify the driving anomalies in some of the pre-KONVOI PWRs where an increase of the noise level was observed. Various research organizations have undertaken the modeling of fuel assembly vibrations using either commercial codes typically working in the time-domain or using in-house tools working in the time- or frequency-domains. As most of these codes use nodal methods and X-Y assembly-homogenized cross sections, the modeling of fuel assembly vibrations using these nodal codes is not straightforward. More specifically, introducing the variations of the cross sections only in the nodes corresponding to the moving fuel assembly does not allow proper modeling of the vibrations and their inherent out-of-phase character.

The purpose of this paper is to investigate the impact of cross section homogenization on the modeling of fuel assembly vibrations. We perform a comparative analysis between the classical approach that uses nodal methods and a novel approach where cross section homogenization is performed at the pin level in an attempt to better reproduce local intra-nodal perturbations. The different approaches based on the ϵ/d model are presented in Section 2. The calculation methodology together with the tools and the simplified core design used in the study, are described in Section 3. The corresponding results are discussed in Section 4, followed by the conclusions in Section 5.

2. Noise analysis approach

Fuel assembly vibrations can be seen as the collective movement of the fuel pins belonging to a given vibrating fuel assembly. Although fuel pins can also move individually and independently from each other within the same fuel assembly, only the collective movement of the fuel pins is studied in this paper. The methods presented hereafter can be easily extended to any type of fuel pin vibrations. For the sake of simplicity and of illustration, vibrations of infinitely-long fuel pins in a one-dimensional system are considered hereafter, i.e. only the radial movement of fuel assemblies along one preferred direction is dealt with. Due to the 1-D nature of the model, the possible axial shape of the vibrations of the fuel assemblies is disregarded. The system being considered is exemplified in Figure 1, where only a limited number of fuel pins are explicitly represented and a

displacement $\epsilon(t)$ along the x-direction is shown.

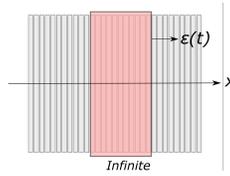


Figure 1: Representation of the one-dimensional system being considered (only a few pins are represented). The grey rectangles represent the fuel pins, whereas the pink rectangle represent the vibrating fuel assembly.

The challenge with modeling fuel assembly vibrations is twofold. First, the displacements are in the range of sub-millimeters, a scale that is much smaller than the coarse spatial mesh used in commercial neutron transport codes. A refined mesh in the vicinity of the perturbation would therefore be necessary to properly model such a highly localized perturbation. Second, the computational grid used by typical neutron transport codes is fixed. The difficulty here lies with the representation of a moving structure on such a fixed computational grid. Modeling approaches other than the classical ones are therefore called for. Several alternatives are possible. Here, we present the ϵ/d model with two approaches in more detail.

2.1. ϵ/d model at the pin cell level

One way to model a vibrating fuel assembly is to model each vibrating fuel pin surrounded by moderator as the juxtaposition of three homogeneous regions, Region I, II and III. This modeling strategy is illustrated in Figure 2.

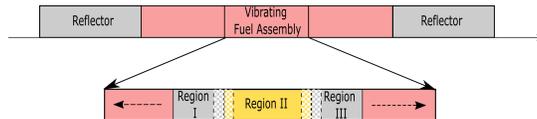


Figure 2: Noise source modeling strategy for the pin-wise approach. A typical vibrating fuel pin surrounded by moderator is shown in the enlarged image. Region I and III represent the moderator and Region II represents the fuel; Dashed boxes represent the mesh nodes (not all are shown here). The noise source is defined on the shaded mesh nodes.

Considering for the time being Region II and Region III, the spatial distribution of the static macroscopic neutron cross section for a reaction type α in the energy group g can be represented as [4]:

$$\Sigma_{\alpha,g}(x) = [1 - \Theta(x - b)]\Sigma_{\alpha,g,II} + \Theta(x - b)\Sigma_{\alpha,g,III} \quad (1)$$

where $\Theta(x - b)$ is the Heaviside function, i.e.

$$\Theta(x - b) = 0 \text{ if } x < b, \Theta(x - b) = 1 \text{ if } x \geq b \quad (2)$$

In Eq. 1, $\Sigma_{\alpha,g,II}$ and $\Sigma_{\alpha,g,III}$ represent the macroscopic cross sections of Region II and III, respectively. In case of vibrations of Region II with respect to Region III with a dimensionless displacement $\epsilon(t)$, the position of the boundary between Regions II and III is time-dependent, and is given as $b(x, t) = b_0 + \epsilon(t)$, where b_0 represents the static position of the boundary between Regions II and III. Note that the parameter $\epsilon(t)$ is relative to the fixed reference position b_0 . Putting $b(x, t)$ into Eq. 1, and using a first-order Taylor expansion, one obtains:

$$\Sigma_{\alpha,g}(x, t) = [1 - \Theta(x - b_0)]\Sigma_{\alpha,g,II} + \Theta(x - b_0)\Sigma_{\alpha,g,III} + \epsilon(t)\delta(x - b_0)[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}] \quad (3)$$

Since the static macroscopic cross section (i.e. when $\epsilon(t) = 0$) is given as:

$$\Sigma_{\alpha,g,0}(x) = [1 - \Theta(x - b_0)]\Sigma_{\alpha,g,II} + \Theta(x - b_0)\Sigma_{\alpha,g,III} \quad (4)$$

the noise source corresponding to fluctuations of the position of the boundary between Regions II and III is consequently expressed as:

$$\delta\Sigma_{\alpha,g}(x, t) = \epsilon(t)\delta(x - b_0)[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}] \quad (5)$$

or in the frequency-domain

$$\delta\Sigma_{\alpha,g}(x, \omega) = \epsilon(\omega)\delta(x - b_0)[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}] \quad (6)$$

In case of vibrations, Region II is displaced by $\epsilon(t)$ compared to the equilibrium position b_0 of the boundary between Regions II and III, and the same displacement $\epsilon(t)$ occurs with respect to the equilibrium position a_0 of the boundary between Regions I and II. A similar treatment of the vibrations with respect to a_0 as the one presented above with respect to b_0 leads to the following final expression for the noise source in the frequency-domain:

$$\delta\Sigma_{\alpha,g}(x, \omega) = \epsilon(\omega)\delta(x - a_0)[\Sigma_{\alpha,g,I} - \Sigma_{\alpha,g,II}] + \epsilon(\omega)\delta(x - b_0)[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}] \quad (7)$$

As can be seen in the expression above, the noise source corresponding to the vibrations of a fuel assembly with respect to its two neighbors is therefore described by two Dirac-like perturbations located at the outer static boundary of the vibrating fuel pin. Based on the model represented by Eq. 7, the induced neutron noise can be estimated in the frequency domain using the Green's function technique, which allows writing [5]:

$$\begin{bmatrix} \delta\phi_1(x, \omega) \\ \delta\phi_2(x, \omega) \end{bmatrix} = \begin{bmatrix} \int [G_{1 \rightarrow 1}(x, x', \omega)S_1(x', \omega) + G_{2 \rightarrow 1}(x, x', \omega)S_2(x', \omega)]dx' \\ \int [G_{2 \rightarrow 1}(x, x', \omega)S_1(x', \omega) + G_{2 \rightarrow 2}(x, x', \omega)S_2(x', \omega)]dx' \end{bmatrix} \quad (8)$$

with

$$\begin{bmatrix} S_1(x', \omega) \\ S_2(x', \omega) \end{bmatrix} = \phi_r(x') \delta \Sigma_r(x', \omega) + \phi_a(x') \begin{bmatrix} \delta \Sigma_{a,1}(x', \omega) \\ \delta \Sigma_{a,2}(x', \omega) \end{bmatrix} + \phi_f(x', \omega) \begin{bmatrix} \delta \nu \Sigma_{f,1}(x', \omega) \\ \delta \nu \Sigma_{f,2}(x', \omega) \end{bmatrix} \quad (9)$$

where the expressions for Σ_r , Σ_a and Σ_f can be found in [4].

2.2. ϵ/d model at the fuel assembly level

The ϵ/d model can also be applied to the system after each fuel assembly has been spatially homogenized. The noise source can either be modeled as two Dirac-like perturbations introduced at the boundaries between the vibrating fuel assembly and its two neighbors or as a coarse mesh approximation of these two noise sources. The latter would be consistent with the strategy of coarse mesh simulations, where the modeling of perturbations and of their effect is carried out at the node level, which represents the smallest mesh size on which the static cross sections and the dynamic ones can be introduced. The modeling of the noise source according to this approach is illustrated in Figure 3.

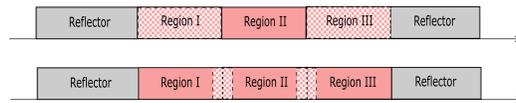


Figure 3: Noise source modeling strategy for the ‘assembly-approximated’ (top figure) and the ‘boundary-localized’ (bottom figure) nodal approaches. Region I and III represent the surrounding fuel assembly and Region II represents the vibrating fuel assembly; Dashed boxes represent the mesh nodes (not all are shown here). The noise source is defined on the shaded mesh nodes.

Another modeling alternative, called the Feinberg-Galanin-Williams (FGW) model [6] in the weak absorber formulation of Pázsit [7] could also be used to model vibrating thin structures. A more exact representation of the movement of fuel pins could also be carried out using a super-fine mesh, as shown in another work [8]. Such alternative modeling strategies are not considered in this study.

3. Calculation methodology

A description of the calculation route along with the tools used for the neutron noise analysis is given here. The modeling of the fuel assembly vibrations is done in two steps. First, two-group homogenized macroscopic cross sections are generated with the Monte-Carlo code SERPENT2 [9] and second, neutron noise sources derived from these homogenized cross sections are fed to CORE SIM [10] to calculate the induced neutron noise.

3.1. Cross section generation with SERPENT2

SERPENT2 is a 3-D continuous-energy reactor physics Monte Carlo transport and burnup calculation code developed at VTT Technical Research Centre in Finland. It reads the continuous-energy cross sections from the JEFF-3.1 library. Group constants are calculated by first homogenizing the geometry using an intermediate multi-group structure with g -groups, and then collapsed into few-group structure with G -groups using infinite-medium and B1 leakage-corrected neutron spectra.

A simplified 2-D core design representative of an experimental light water reactor is used for the generation of the cross sections needed in the study. The core has 3x3 fuel assemblies, each assembly containing 17x17 pins fueled with UOX and surrounded by water. The vibrating fuel assembly is chosen to have a slightly lower enrichment (2.5% UOX) compared to the rest of the fuel assemblies (3.7% UOX). This is done to ensure the changes in cross sections between the neighboring fuel assemblies are higher than their statistical fluctuations, when generating the cross sections with a Monte-Carlo code. In this work, diffusion coefficient, absorption cross section, removal cross section derived from the scattering cross section and ν -weighted fission cross sections in fast ($G=1$) and thermal ($G=2$) energy group with a cut-off at 0.625 eV are the input macroscopic data. The removal cross section is defined as the isotropic down-scattering cross section minus the isotropic up-scattering cross section weighted with the ratio between the thermal and the fast neutron fluxes. The parameter ν is the average number of neutrons emitted per fission. For the pin calculations, the cross section for the fuel and moderator are calculated separately, at the pin level, unlike the homogenization of the entire fuel assembly for calculation of nodal cross sections.

3.2. Neutron noise calculation with CORE SIM

CORE SIM is a MATLAB-based neutronics solver of the 2-group neutron noise diffusion equations in the frequency domain (i.e. for stationary fluctuations). It requires a uniform mesh distribution, static cross sections, noise sources and point-kinetics data of the reactor core as input. In this work, CORE SIM uses sets of cross sections and kinetic parameters obtained from SERPENT2. An explicit modeling of the mechanical vibrations of the fuel assemblies is not possible in CORE SIM. However, such perturbations can be expressed as variations of macroscopic cross sections. In particular, the effect of the vibration of a fuel assembly is introduced indirectly via neutron noise sources defined as the differences in cross sections between the vibrating and the neighboring assemblies, as discussed in Section 2. The removal cross section, the fast absorption cross section, and fast and thermal fission cross section contribute to the fast noise source, while the thermal noise source contains contributions from the removal cross section and the thermal absorption (thermal fission included) cross section, only.

For that purpose, a 1-D reactor model of size 130 cm is considered. In order to model sub-millimeter displacements of the fuel assembly, a fine mesh is required. Here, the choice of a simplified 1-D reactor model is made to ensure the calculation time and memory requirement for the simulations are reasonable. In the case of the ‘nodal approach’, the core is assumed to be made of three regions: a central region representing one fuel assembly of size 21.42 cm surrounded by two identical homogeneous regions representing all other fuel assemblies, and a reflector of thickness 32.88 cm surrounding the active core. In the case of the ‘pin approach’, the fuel assemblies are modeled heterogeneously, so that fuel and moderator regions are characterized by two-region

cell-type cross sections. The cross sections for both the nodal and pin approaches are derived from the SERPENT2 calculations of the 2-D core. All simulations are performed in the frequency domain at a frequency of 1 Hz, which corresponds to typical eigenfrequencies of the first mode of vibration. The induced neutron noise is estimated using a 1-D version of CORE SIM. The spatial mesh size, set uniformly to 0.03 cm, is chosen such that it is smaller than the maximum displacement seen by the vibrating assemblies. Specifically, for pin-level calculations, it must be ensured that the mesh nodes do not contain two different materials, and mesh boundaries coincide with the edges of the fuel and moderator elements.

It is recognized that a solver based on transport theory rather than diffusion theory would be better suited to address this problem. In this regard, development work is going on a transport tool, which is based on a similar philosophy as CORE SIM [11].

4. Results and discussion

In this section, the results corresponding to the modeling of a single fuel assembly vibrations in a small light-water reactor using the ϵ/d model at both nodal and pin-level are reported.

4.1. Case 1: Nodal calculations

In the first set of simulations, Case 1a, referred to as ‘boundary-localized’, an average of the two noise sources defined as the difference of the static macroscopic cross sections between the central fuel assembly and the surrounding two (identical, 21.42-cm thick) regions is introduced at the corresponding boundaries. Each of these perturbations at the boundaries is in fact located in the two fine-mesh nodes (of 0.03 cm) that are respectively on the left and on the right side of the boundaries. In the second set of simulations, Case 1b, referred to as ‘assembly-approximated’, two noise sources are also defined in a similar manner. However, they are introduced across the entire volume of the two 21.42 cm-thick fuel assemblies neighboring the central moving fuel assembly, with the moving fuel assembly remaining unperturbed. The latter approach corresponds to the modeling strategy used in nodal codes not specifically made for representing fuel assembly vibrations.

Results are given in Figure 4 for the amplitude and phase of the induced neutron noise, normalized to the respective volume-averaged neutron noise strengths. As can be noticed, the deviation in phase is mostly negligible between the two approaches. The peculiar change of the thermal phase evaluated with the ‘boundary-localized’ nodal approach is a result of the combined effect of the fast and the thermal noise sources. The general behavior is driven by the fast neutron noise source, except at the boundary of the vibrating fuel assembly, where the perturbations are introduced according to the approach (see Figure 4 and Figure 5(b)). In the vicinity of the perturbation, a drop of the phase of the thermal noise is predicted. This is due to the effect of the thermal noise source on the induced thermal noise, which is larger than the one associated with the fast noise source (see Figure 5(d) and Figure 5(h)). On the other hand, for the amplitude in Figures 4 & 5, the peaks coincide with the boundaries of the vibrating fuel assembly in the ‘boundary-localized’ approach. The deviation between Case 1a and Case 1b is relatively larger in the close vicinity of the perturbations. This is explained by the local component of the induced neutron noise in two-group theory. A few mean free paths from the introduced perturbations, the two modeling strategies lead to closer results.

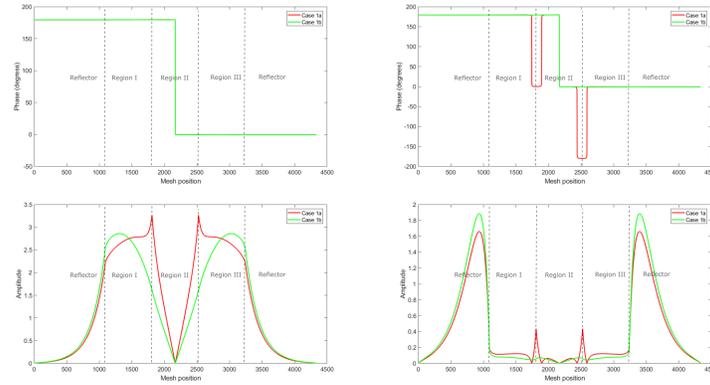


Figure 4: Comparisons of the phase (top figures) and amplitude (bottom figures) of the induced neutron noise between the ‘boundary-localized’ (red) and the ‘assembly-approximated’ (green) approach when both fast and thermal noise sources are introduced. Dashed lines represent the interfaces in the core. They are added as guide for the eye.

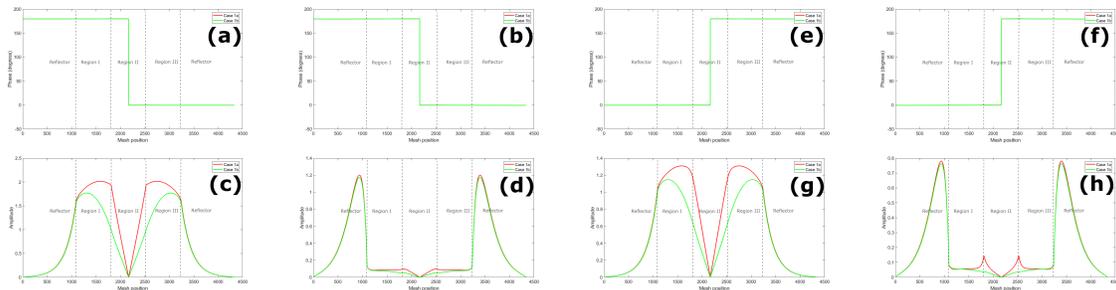


Figure 5: Comparisons of the phase (top figures) and amplitude (bottom figures) of the induced neutron noise between the ‘boundary-localized’ (red) and the ‘assembly-approximated’ (green) approach when fast noise sources (a-d) and thermal noise sources (e-h) are introduced separately. Dashed lines represent the interfaces in the core. They are added as guide for the eye.

4.2. Case 2: Pin-level calculations

For the pin-level calculations, noise sources defined as the difference of the static macroscopic cross sections between the fuel pins and the surrounding moderator regions for each of the 17 fuel pins in the perturbed fuel assembly, are introduced at the corresponding boundaries. The perturbations are actually introduced in the immediate vicinity of each boundary, over two neighboring meshes of 0.03 cm, similarly to Case 1b. An expected out-of-phase behavior (not represented here for the sake of brevity) is seen in the neutron noise. The amplitude is distributed symmetrically around the vibrating fuel assembly.

Figure 6 shows a comparison of the induced neutron noise amplitude between the two nodal ap-

proaches and the pin approach when both the fast and thermal noise sources are introduced. The neutron noise estimated from the nodal and pin approaches show similar profiles away from the perturbation. However, the reflector region sees a slight deviation in the induced thermal neutron noise between the two approaches. Case 1b predicts a slightly higher amplitude of the neutron noise in the reflector region because it defines the perturbation in a coarse manner over the entire surrounding fuel assemblies. In Case 1a & 2, both approaches show some noticeable peaks at the location of the perturbations as expected, because they consider the localized character of the neutron noise source. In Case 2, the neutron noise “oscillates” inside the vibrating fuel assembly since the pin approach considers a superposition of the local effects due to the movement of each fuel rod. It is also seen that the thermal neutron noise in the vibrating fuel assembly is slightly larger than the one in the nodal approaches.

The thermal neutron noise is of particular interest since usual neutron detectors in power reactors are mostly sensitive to the thermal flux. Considering this aspect, the three approaches lead to comparable results as long as detectors are located inside the active core away from the perturbation, even though the three predictions of the fast neutron noise may show differences. In the vicinity of the vibrating fuel assembly, the ‘boundary-localized’ nodal approach seems to capture local neutron noise without the need of the complexity of a pin-by-pin model.

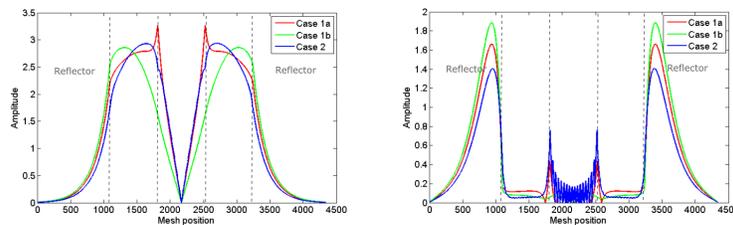


Figure 6: Comparisons of the amplitude of the induced neutron noise between the nodal and pin-wise approach when both fast and thermal noise sources are introduced at the same time. Dashed lines represent the interfaces in the core. They are added as guide for the eye.

5. CONCLUSIONS

In this work, a simple 1D CORESIM-based model of neutron noise sources resulting from fuel assembly vibrations was presented, and the impact of cross section homogenization on this particular perturbation was studied. This study demonstrates that nodal codes can faithfully represent the collective and coherent movement of the fuel pins belonging to a fuel assembly at the node level. Both nodal approaches lead to essentially identical results sufficiently away from the perturbation. However, close to the perturbation, the two approaches provide rather different responses, and it is suggested that the ‘boundary-localized’ nodal approach should be preferred over a classical node-wide approach if the code allows introducing perturbations at the boundary of two adjacent nodes, possibly in combination with neutron detector signals located in the vicinity. The ‘boundary-localized’ nodal approach appears to be capable of capturing local noise information (as a pin-wise approach would do), without requiring an ultra-fine mesh.

Since the vibrations of a fuel assembly are modeled as differences between macroscopic cross

sections, the relative neutron noise calculations are highly sensitive to the details of the core configuration, which makes it challenging to realistically model fuel assembly vibrations. Moving from 1-D to more realistic 3-D models and larger reactors would result in additional complications, such as asymmetric induced neutron noise. One of the limitations of this study is that the whole fuel assembly, implying all fuel rods within it, are assumed to move with the same frequency of 1 Hz. In reality though, the vibrations do not necessarily follow a uniform pattern. They may not vibrate with the same frequency at the same time and with the same amplitude and phase. Also, all simulations were carried out irrespective of actual displacements of the fuel assemblies. However, because of the assumption of linearity in the current noise model, the calculated induced neutron noise can be scaled with respect to the displacements, though it would be interesting to calculate realistic displacements. Therefore, in future work, it would be interesting to extend these noise modeling methods to a larger variety of perturbation initiators.

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