CORTEX

Core monitoring techniques and experimental validation and demonstration

Neutron noise simulation for development and testing

CORTEX Workshop

Advanced signal processing methods and learning methodologies applied to the monitoring of NPP reactor conditions

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Introduction

- Neutron noise diagnostics requires establishing relationships between neutron detectors and possible perturbations

- Since all neutron transport codes use nuclear macroscopic cross-sections as input, need to convert "physical" perturbations into perturbations of macroscopic cross-sections

➢ Use of expert opinion for expressing such perturbations of macroscopic cross-sections, possibly supplemented by other modelling tools (e.g. fluid-structure modelling tools)

➢ Noise source modelling

Introduction

- Once the noise source is modelled, need to estimate the response of the neutron flux to the applied perturbation

➢Could be done using the neutron transport equation (Boltzmann equation):

$$
\begin{aligned}
&\frac{1}{v(E)}\frac{\partial}{\partial t}\psi(\mathbf{r},\mathbf{\Omega},E,t) \\
&= -\mathbf{\Omega}\cdot\nabla\psi(\mathbf{r},\mathbf{\Omega},E,t) - \Sigma_t(\mathbf{r},E,t)\psi(\mathbf{r},\mathbf{\Omega},E,t) \\
&+ \int_{(4\pi)}\int_0^{\infty}\Sigma_s(\mathbf{r},\mathbf{\Omega}'\rightarrow\mathbf{\Omega},E'\rightarrow E,t)\psi(\mathbf{r},\mathbf{\Omega}',E',t)\,d^2\mathbf{\Omega}'dE' \\
&+ \frac{1}{4\pi}\int_{-\infty}^{t}\int_0^{\infty}\nu(E')\Sigma_f(\mathbf{r},E',t')\phi(\mathbf{r},E',t')\left[(1-\beta)\chi^p(E)\delta(t-t') + \sum_{i=1}^{N_d}\chi_i^d(E)\lambda_i\beta_i e^{-\lambda_i(t-t')}\right]dt'dE'
\end{aligned}
$$

Introduction

- Neutron noise transport equation = integro-differential equations in the multi-dimensional phase space $\left(\mathbf{r}, \mathbf{\Omega}, E, t\right)$

➢Simpler formalisms usually used for modelling nuclear reactor cores, such as the multi-group diffusion approximation:

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t}\left(\mathbf{r},t\right)$$
$$= \mathbf{\nabla}\cdot\left[D_g\left(\mathbf{r},t\right)\mathbf{\nabla}\phi_g\left(\mathbf{r},t\right)\right] - \Sigma_{t,g}\left(\mathbf{r},t\right)\phi_g\left(\mathbf{r},t\right)$$
$$+\sum_{g'=1}^{G}\Sigma_{s0,g'\to g}\left(\mathbf{r},t\right)\phi_{g'}\left(\mathbf{r},t\right) + \left(1-\beta\right)\chi_g^p\sum_{g'=1}^{G}\nu_{g'}\Sigma_{f,g'}\left(\mathbf{r},t\right)\phi_{g'}\left(\mathbf{r},t\right) + \sum_{i=1}^{N_g}\lambda_i\chi_{i,g}^d C_i\left(\mathbf{r},t\right)$$

with

$$\frac{\partial C_i}{\partial t}\left(\mathbf{r},t\right) = \beta_i\sum_{g'=1}^{G}\nu_{g'}\Sigma_{f,g'}\left(\mathbf{r},t\right)\phi_{g'}\left(\mathbf{r},t\right) - \lambda_i C_i\left(\mathbf{r},t\right), i = 1,\ldots,N_d$$

Modelling approaches

- Different approaches possible:
  - Time-domain modelling

    Advantages:
    - Existing time-domain codes could be used
    - Non-linear effects inherently accounted for
    - Thermal-hydraulic feedback automatically taken into account

    Disadvantages:
    - Lengthy calculations
    - Challenging to get a highly accurate solution for the noise
    - Codes originally not developed for that purpose
    - Lack of verification and validation for noise analyses

Modelling approaches

- Different approaches possible:
  - Frequency-domain modelling

    Time-domain equations transformed into frequency-domain equations according to the following procedure:

    - Splitting between mean values and fluctuations
    - Linear theory used because of the smallness of the fluctuations
    - Fourier-transform of the balance equations for the dynamical part only

Modelling approaches

- Different approaches possible:
  - Frequency-domain modelling

    Advantages:
    - Codes specifically developed for noise analysis, thus usually fully verified (validated?)
    - Highly accurate noise solution
    - Usually high flexibility in the modelling
    - Very fast calculations

    Disadvantages:
    - No commercial code available
    - Possible linear effects disregarded
    - Thermal-hydraulic feedback generally not taken into account (but could be)

Modelling tools used in CORTEX

- Time-domain codes:

  Existing codes:
  - SIMULATE-3K (diffusion): commercial code developed by Studsvik Scandpower
  - DYN3D (diffusion): "research" code developed by HZDR
  - QUABBOX/CUBBOX (diffusion): "research" code developed by GRS
  - PARCS (diffusion): "research" code developed by Purdue University/the University of Michigan

  New codes being developed/extended:
  - FEMFUSSION (diffusion): "research" code developed by UPV
  - APOLLO3® (transport): "research" code developed by CEA

Modelling tools used in CORTEX

- Frequency-domain codes:

  Existing codes:
  - CORE SIM (diffusion): "research" code developed by Chalmers – freely available

  New codes being developed:
  - CORE SIM+ (diffusion): "research" code developed by Chalmers
  - Sn-based solver (deterministic transport): "research" code developed by Chalmers
  - APOLLO3® (deterministic transport): "research" code developed by CEA
  - MCNP (probabilistic transport): developments made by KURRI
  - TRIPOLI4® (probabilistic transport): "research" code developed by CEA
  - SERPENT2/MCNP (probabilistic transport): developments made by Chalmers

Modelling tools used in CORTEX

- Most of the new tools being developed focus on transport, so that the validity of diffusion-based solvers can be better assessed

Noise source modelling

- Different scenarios investigated in CORTEX:
  - “Absorber of variable strength”
  - “Vibrating absorber”
  - Axially-travelling perturbations
  - Fuel assembly vibrations
  - Core barrel vibrations

"Absorber of variable strength" type of noise source

- "Absorber of variable strength" = localized perturbation of which its amplitude varies in time at a fixed position

- Induced neutron noise given by the following balance equation (2-group diffusion theory):

$$\left\{\nabla\cdot\left[\mathbf{D}\left(\mathbf{r}\right)\nabla\right]+\Sigma_{dyn}\left(\mathbf{r},\omega\right)\right\}\times\begin{bmatrix}\delta\phi_1\left(\mathbf{r},\omega\right)\\ \delta\phi_2\left(\mathbf{r},\omega\right)\end{bmatrix}$$

$$=\phi_r\left(\mathbf{r}\right)\delta\Sigma_r\left(\mathbf{r},\omega\right)+\phi_a\left(\mathbf{r}\right)\begin{bmatrix}\delta\Sigma_{a,1}\left(\mathbf{r},\omega\right)\\ \delta\Sigma_{a,2}\left(\mathbf{r},\omega\right)\end{bmatrix}+\phi_f\left(\mathbf{r},\omega\right)\begin{bmatrix}\delta\upsilon\Sigma_{f,1}\left(\mathbf{r},\omega\right)\\ \delta\upsilon\Sigma_{f,2}\left(\mathbf{r},\omega\right)\end{bmatrix}$$

"Absorber of variable strength" type of noise source

- In case of a point-like source:

$$\left[\nabla_{\mathbf{r}} \cdot \left[\mathbf{D}\left(\mathbf{r}\right)\nabla_{\mathbf{r}}\right] + \Sigma_{dyn}\left(\mathbf{r},\omega\right)\right] \times \begin{bmatrix} G_{g\to 1}\left(\mathbf{r},\mathbf{r}',\omega\right) \\ G_{g\to 2}\left(\mathbf{r},\mathbf{r}',\omega\right) \end{bmatrix} = \begin{bmatrix} \delta\left(\mathbf{r}-\mathbf{r}'\right) \\ 0 \end{bmatrix}_{g=1} \text{ or } \begin{bmatrix} 0 \\ \delta\left(\mathbf{r}-\mathbf{r}'\right) \end{bmatrix}_{g=2}$$

➢ Green's function

"Absorber of variable strength" type of noise source

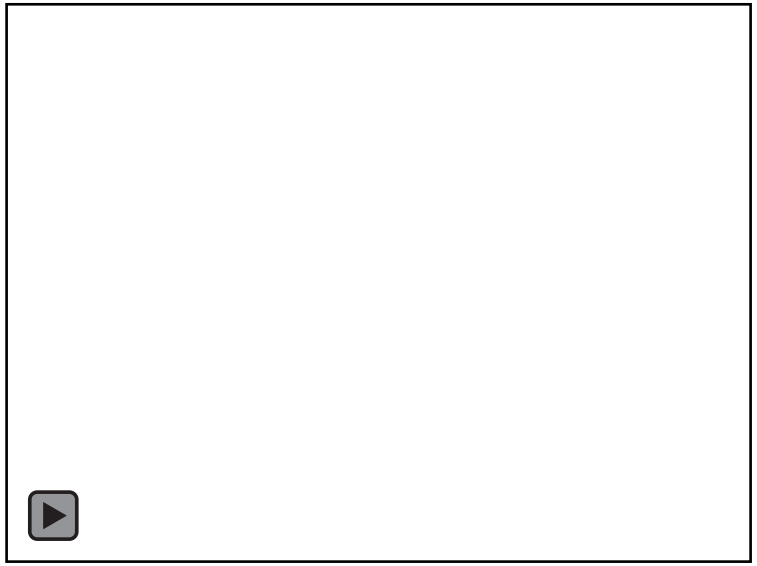
- General solution to the original problem can be given by convolution integrals

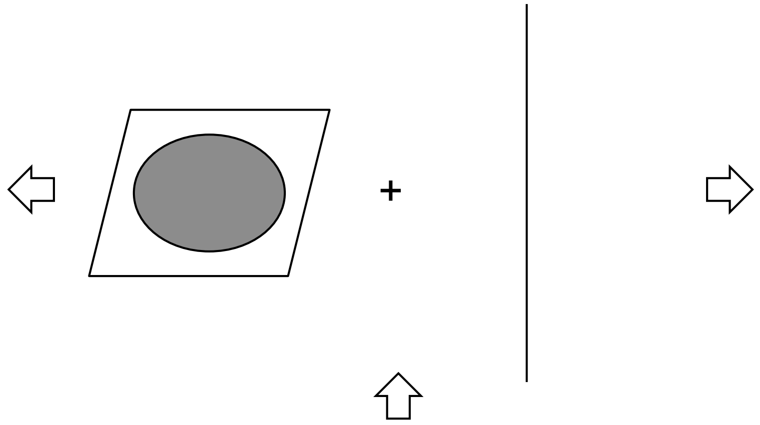
$$\begin{bmatrix} \delta\phi_1(\mathbf{r},\omega) \\ \delta\phi_2(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \int \left[G_{1\to1}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to1}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \\ \int \left[G_{1\to2}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to2}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \end{bmatrix}$$

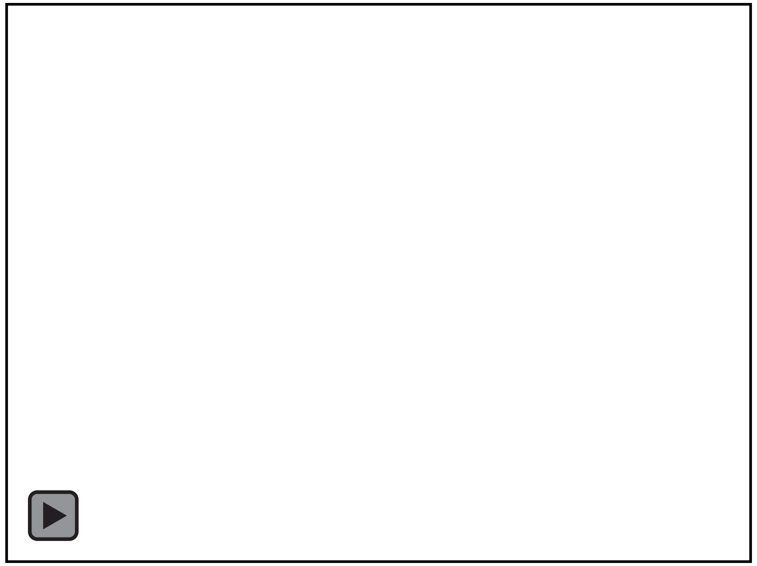
with $$\begin{bmatrix} S_1(\mathbf{r}',\omega) \\ S_2(\mathbf{r}',\omega) \end{bmatrix} = \phi_r(\mathbf{r}') \delta\Sigma_r(\mathbf{r}',\omega) + \phi_a(\mathbf{r}') \begin{bmatrix} \delta\Sigma_{a,1}(\mathbf{r}',\omega) \\ \delta\Sigma_{a,2}(\mathbf{r}',\omega) \end{bmatrix} + \phi_f(\mathbf{r}',\omega) \begin{bmatrix} \delta\upsilon\Sigma_{f,1}(\mathbf{r}',\omega) \\ \delta\upsilon\Sigma_{f,2}(\mathbf{r}',\omega) \end{bmatrix}$$

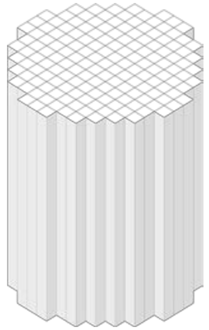
“Absorber of variable strength” type of noise source

- Example of a localized “absorber of variable strength” @ 1kHz









"Vibrating absorber" type of noise source

- Lateral movement of the absorber represented as (weak absorber):

$$\delta\Sigma_{a,2}\left(\mathbf{r},t\right)=\gamma\theta\left(z-z_0\right)\left[\delta\left(\mathbf{r}_{xy}-\mathbf{r}_{p,xy}-\boldsymbol{\varepsilon}\left(t\right)\right)-\delta\left(\mathbf{r}_{xy}-\mathbf{r}_{p,xy}\right)\right]$$

- A first-order Taylor expansion of the noise source would give for the induced neutron noise (in the frequency-domain):

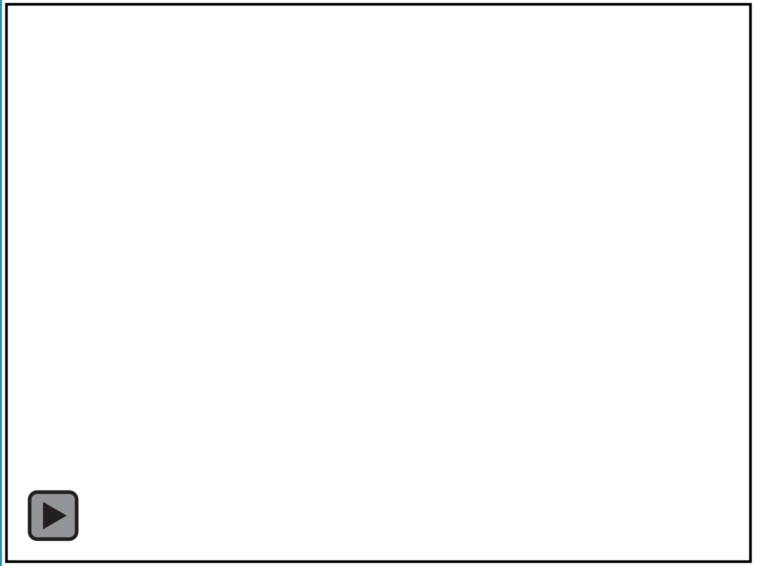
$$\delta\phi_g\left(\mathbf{r},\omega\right)=-\gamma\boldsymbol{\varepsilon}\left(\omega\right)\cdot\boldsymbol{\delta\varphi}_g\left(\mathbf{r},\omega\right)$$

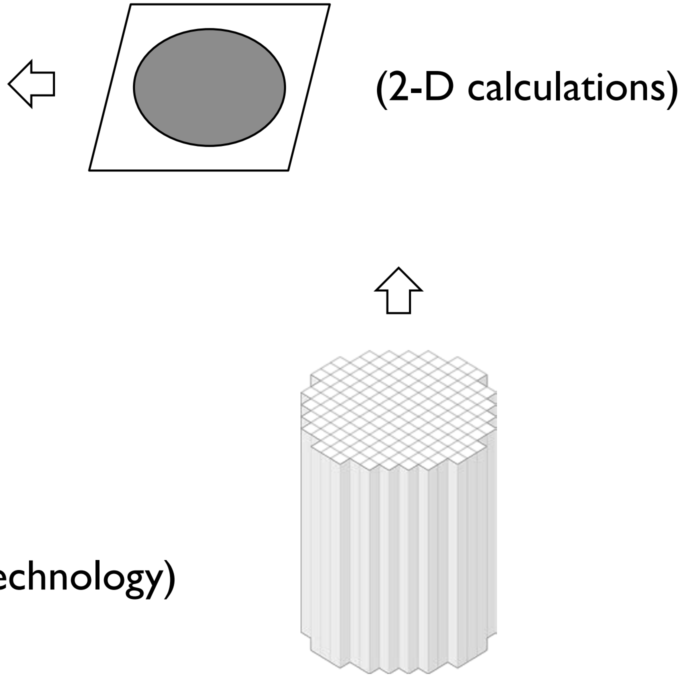
with

$$\boldsymbol{\delta\varphi}_g\left(\mathbf{r},\omega\right)=\nabla_{\mathbf{r}_{p,xy}}\hat{G}_{2\to g}\left(\mathbf{r},\mathbf{r}_{p,xy},\omega\right)$$

“Vibrating absorber” type of noise source

- Example of a vibrating control rod @ 0.2 Hz





Axially-travelling perturbations

- Noise source represented in the time-domain as:

$$\delta\Sigma_{rem}\left(\mathbf{r},t\right) \equiv \delta\Sigma_{rem}\left(x,y,z,t\right)$$
$$= \begin{cases} 0,\ \text{if } \left(x,y\right) \neq \left(x_0,y_0\right) \\ 0,\ \text{if } \left(x,y\right) = \left(x_0,y_0\right) \text{ and } z < z_0 \\ \delta\Sigma_{rem}\left(x_0,y_0,z_0,t-\dfrac{z-z_0}{v}\right),\ \text{if } \left(x,y\right) = \left(x_0,y_0\right) \text{ and } z \geq z_0 \end{cases}$$

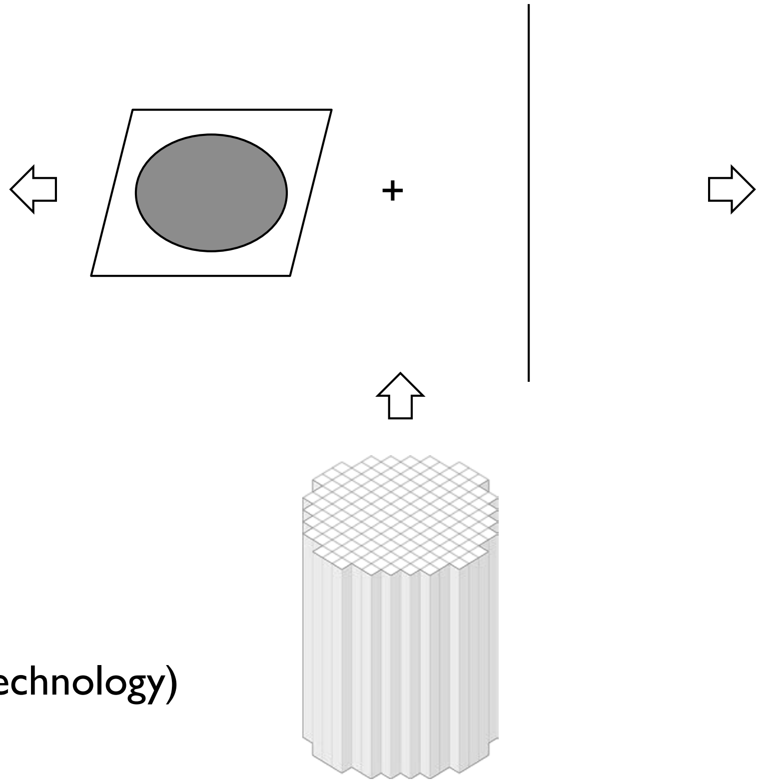
Axially-travelling perturbations

- Noise source represented in the frequency-domain as:

$$\delta\Sigma_{rem}\left(\mathbf{r},\omega\right) \equiv \delta\Sigma_{rem}\left(x,y,z,\omega\right)$$
$$= \begin{cases} 0, \text{ if } \left(x,y\right) \neq \left(x_0,y_0\right) \\ 0, \text{ if } \left(x,y\right) = \left(x_0,y_0\right) \text{ and } z < z_0 \\ \delta\Sigma_{rem}\left(x_0,y_0,z_0,\omega\right)\exp\left[-\dfrac{i\omega\left(z-z_0\right)}{v}\right], \text{ if } \left(x,y\right) = \left(x_0,y_0\right) \text{ and } z \geq z_0 \end{cases}$$

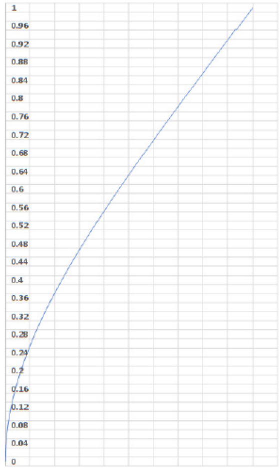
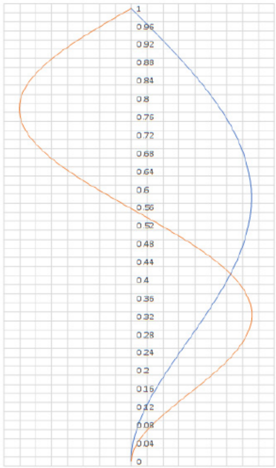
Axially-travelling perturbations

- Example of a travelling perturbation @ 1Hz



Fuel assembly vibrations

- Different possible axial vibration modes for fuel assemblies:

	Cantilevered beam	Simply supported on both sides	Cantilevered beam and simply supported
Axial shape of the displacement $d\left(z,t\right)$ in arbitrary units as a function of the relative core elevation z		 first mode in blue, second mode in orange	 first mode in blue, second mode in orange
Oscillation frequency	Ca. 0.6 – 1.2 Hz	Ca. 0.8 – 4 Hz for the first mode Ca. 5 – 10 Hz for the second mode	Ca. 0.8 – 4 Hz for the first mode Ca. 5 – 10 Hz for the second mode

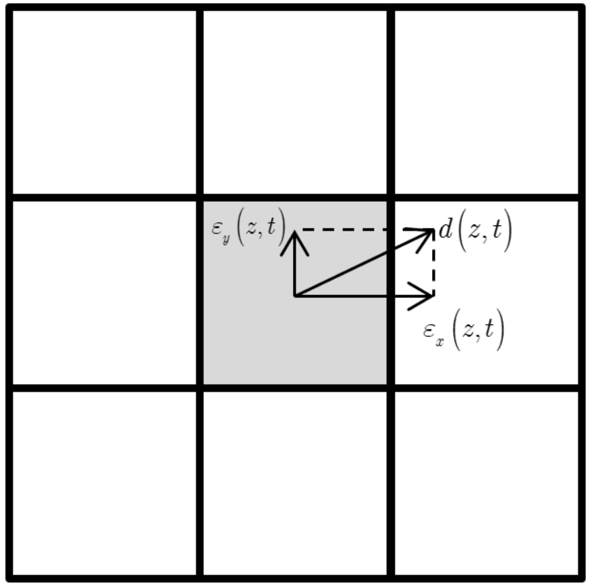
Fuel assembly vibrations

- Fuel assembly vibrations described at the pin level:
  - Can be modelled as "vibrating absorbers"
  - Can be modelled as "absorbers of variable strength" !

- Fuel assembly vibrations at the nodal level can only be modelled as "absorber of variable strength" !

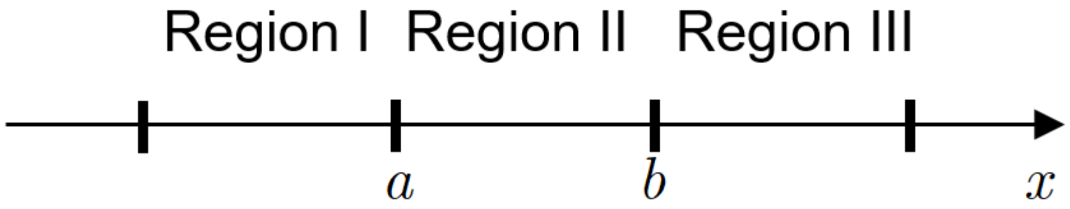
Fuel assembly vibrations

- Lateral vibrations represented as:



Fuel assembly vibrations

- In e.g. the x -direction, one has:



with static cross-section between Regions II and III given as:

$$\Sigma_{\alpha,g}^{x}\left(x\right)=\left[1-\Theta\left(x-b\right)\right]\Sigma_{\alpha,g,II}+\Theta\left(x-b\right)\Sigma_{\alpha,g,III}$$

Fuel assembly vibrations

- For a time-dependent boundary:

$$b(z,t) = b_0 + \varepsilon_x(z,t)$$

one obtains after a first-order Taylor expansion in the time-domain:

$$\Sigma_{\alpha,g}^{x}(x,z,t)$$
$$= \left[1 - \Theta(x - b_0)\right]\Sigma_{\alpha,g,II} + \Theta(x - b_0)\Sigma_{\alpha,g,III} + \varepsilon_x(z,t)\delta(x - b_0)\left[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}\right]$$

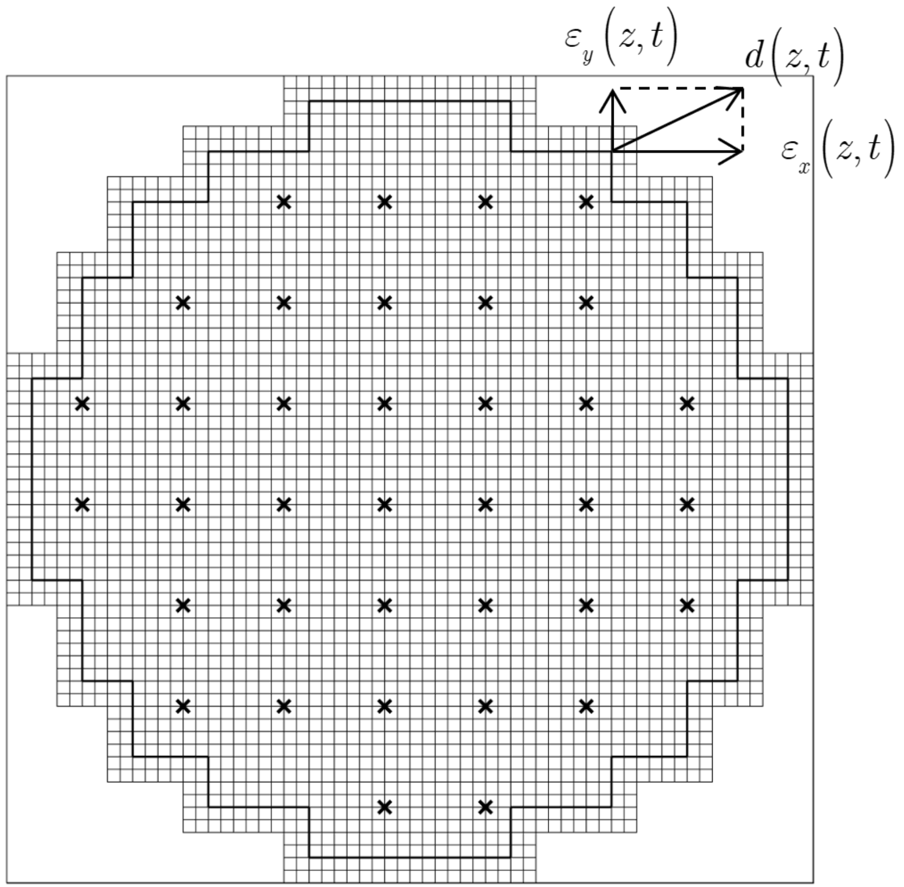
➢ Noise source in the frequency-domain:

$$\delta\Sigma_{\alpha,g}^{x}(x,z,\omega) = \varepsilon_x(z,\omega)\delta(x - b_0)\left[\Sigma_{\alpha,g,II} - \Sigma_{\alpha,g,III}\right]$$

➢ Point-like source!

Pendular core barrel vibrations

- Core barrel vibrations can be seen as a relative displacement of the active core with respect to the reflector:



Pendular core barrel vibrations

- Same technique as for fuel assembly vibrations can be used:

$$\delta\Sigma^{x}_{\alpha,g}\left(x,z\right) = h\left(z\right)\sum_{n}\delta\left(x - x_{n}\right)\left[\Sigma_{\alpha,g,x_n^-} - \Sigma_{\alpha,g,x_n^+}\right]$$

$$\delta\Sigma^{y}_{\alpha,g}\left(y,z\right) = h\left(z\right)\sum_{m}\delta\left(y - y_{m}\right)\left[\Sigma_{\alpha,g,y_m^-} - \Sigma_{\alpha,g,y_m^+}\right]$$

- Point-like source!

Estimation of the induced neutron noise

- Generically, the induced neutron noise is given as (e.g. in 2-group theory):

$$\begin{bmatrix} \delta\phi_1(\mathbf{r},\omega) \\ \delta\phi_2(\mathbf{r},\omega) \end{bmatrix} = \begin{bmatrix} \int \left[G_{1\to 1}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to 1}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \\ \int \left[G_{1\to 2}(\mathbf{r},\mathbf{r}',\omega) S_1(\mathbf{r}',\omega) + G_{2\to 2}(\mathbf{r},\mathbf{r}',\omega) S_2(\mathbf{r}',\omega) \right] d^3\mathbf{r}' \end{bmatrix}$$

with $$\begin{bmatrix} S_1(\mathbf{r}',\omega) \\ S_2(\mathbf{r}',\omega) \end{bmatrix} = \boldsymbol{\phi}_r(\mathbf{r}')\delta\Sigma_r(\mathbf{r}',\omega) + \boldsymbol{\phi}_a(\mathbf{r}') \begin{bmatrix} \delta\Sigma_{a,1}(\mathbf{r}',\omega) \\ \delta\Sigma_{a,2}(\mathbf{r}',\omega) \end{bmatrix} + \boldsymbol{\phi}_f(\mathbf{r}',\omega) \begin{bmatrix} \delta\upsilon\Sigma_{f,1}(\mathbf{r}',\omega) \\ \delta\upsilon\Sigma_{f,2}(\mathbf{r}',\omega) \end{bmatrix}$$

Estimation of the induced neutron noise

… or given as:

$$\delta\phi_g\left(\mathbf{r},\omega\right) = -\gamma\boldsymbol{\varepsilon}\left(\omega\right)\cdot\boldsymbol{\delta\varphi}_g\left(\mathbf{r},\omega\right)$$

with

$$\boldsymbol{\delta\varphi}_g\left(\mathbf{r},\omega\right) = \nabla_{\mathbf{r}_{p,xy}}\hat{G}_{2\to g}\left(\mathbf{r},\mathbf{r}_{p,xy},\omega\right)$$

- In essence, only the Green's function is needed

Estimation of the induced neutron noise

- The Green's function can be estimated:
  - Either deterministically
    - Using diffusion theory

$$\left[\nabla_{\mathbf{r}} \cdot \left[\mathbf{D}\left(\mathbf{r}\right)\nabla_{\mathbf{r}}\right] + \Sigma_{dyn}\left(\mathbf{r},\omega\right)\right] \times \begin{bmatrix} G_{g\to 1}\left(\mathbf{r},\mathbf{r}',\omega\right) \\ G_{g\to 2}\left(\mathbf{r},\mathbf{r}',\omega\right) \end{bmatrix} = \begin{bmatrix} \delta\left(\mathbf{r}-\mathbf{r}'\right) \\ 0 \end{bmatrix}_{g=1} \text{ or } \begin{bmatrix} 0 \\ \delta\left(\mathbf{r}-\mathbf{r}'\right) \end{bmatrix}_{g=2}$$

    - Using transport theory

$$\left[\mathbf{\Omega} \cdot \nabla_{\mathbf{r}} + \Sigma_{dyn}\left(\mathbf{r},\mathbf{\Omega},\omega\right)\right] \times \begin{bmatrix} G_{g\to 1}\left(\mathbf{r},\mathbf{\Omega},\mathbf{r}',\mathbf{\Omega}',\omega\right) \\ G_{g\to 2}\left(\mathbf{r},\mathbf{\Omega},\mathbf{r}',\mathbf{\Omega}',\omega\right) \end{bmatrix} = \begin{bmatrix} \delta\left(\mathbf{r}-\mathbf{r}'\right)\delta\left(\mathbf{\Omega}-\mathbf{\Omega}'\right) \\ 0 \end{bmatrix}_{g=1} \text{ or } \begin{bmatrix} 0 \\ \delta\left(\mathbf{r}-\mathbf{r}'\right)\delta\left(\mathbf{\Omega}-\mathbf{\Omega}'\right) \end{bmatrix}_{g=2}$$

Estimation of the induced neutron noise

- The Green's function can also be estimated:
  - Probabilistically:
    - Introducing complex weights in the Monte Carlo solver
    - ➢ Requires modification of the source codes

Estimation of the induced neutron noise

- The Green's function can also be estimated:
  - Probabilistically:
    - Using an equivalence to subcritical problems (demonstrated hereafter on diffusion theory in 2 energy groups):

$$\begin{bmatrix}\delta\phi_1(\mathbf{r},\omega)\\ \delta\phi_2(\mathbf{r},\omega)\end{bmatrix} = \begin{bmatrix}\delta\phi_1^{real}(\mathbf{r},\omega)\\ \delta\phi_2^{real}(\mathbf{r},\omega)\end{bmatrix} + i\begin{bmatrix}\delta\phi_1^{im}(\mathbf{r},\omega)\\ \delta\phi_2^{im}(\mathbf{r},\omega)\end{bmatrix}$$

The coupling to the imaginary (real, respectively) part of the neutron noise when solving for the real (imaginary, respectively) balance equations is treated as additional noise sources

$$\begin{bmatrix}S_1^{real\text{ or }im}(\mathbf{r},\omega)\\ S_2^{real\text{ or }im}(\mathbf{r},\omega)\end{bmatrix} = \begin{bmatrix}\text{Re or Im}\left\{S_1(\mathbf{r},\omega)\right\}\\ \text{Re or Im}\left\{S_1(\mathbf{r},\omega)\right\}\end{bmatrix} + \mathbf{M}\times\begin{bmatrix}\text{Im or Re}\left\{\delta\phi_1(\mathbf{r},\omega)\right\}\\ \text{Im or Re}\left\{\delta\phi_2(\mathbf{r},\omega)\right\}\end{bmatrix}$$

➢ No modification of the source code required

Conclusions and outlook

- Modelling the effect of noise sources can be done in many ways:
  - Time-domain/frequency-domain
  - Diffusion/transport
  - Deterministic/probabilistic

- Taking full advantage of noise analysis requires:
  - A correct modelling of the noise source
  - The estimation of the reactor transfer function
  - Its inversion

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