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Towards a solver based on a discrete ordinate method for reactor neutron noise simulations in the frequency domain

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During operations of a nuclear reactor, neutron flux measurements show fluctuations around the expected mean values. These fluctuations, referred to as neutron noise, may be due to a variety of perturbations such as mechanical vibrations of core internals, disturbances in the coolant flow, etc. From the analysis of the neutron noise, anomalous patterns can be identified at an early stage, so that appropriate actions can be taken before dangerous situations arise. Neutron noise-based monitoring subsequently contributes to enhanced safety. For this purpose, the reactor transfer function, which describes the core response to any possible perturbation, is most often required. The modelling of the reactor transfer function can be based on the Boltzmann equation for the neutron population in the system, while the possible perturbations are expressed in terms of changes in the neutron cross-sections. Most of the past work in this area relies on neutron diffusion theory [1]. However, recent efforts focus on the development of advanced computational capabilities in order to provide more detailed simulations and to assess the limitations of the diffusion approximation for neutron noise applications [2, 3, 4].

The treatment of neutron noise problems in the frequency domain is of particular interest because computationally expensive time-dependent calculations are avoided. The balance equations for the neutron noise in the frequency domain are derived from the following procedure. A critical nuclear system is considered. The perturbations that induce the neutron noise, are modelled as stationary fluctuations of the macroscopic neutron cross-sections, with a prescribed amplitude (much smaller than the mean values of the cross-sections) and angular frequency $\omega = 2\pi f$. In the time-dependent neutron transport equation with a given number of groups of delayed neutrons, the neutron flux, the concentrations of precursors of delayed neutrons and the macroscopic neutron cross-sections are expressed as sums of static mean values (generically denoted as X_0) and time-dependent fluctuating values (generically denoted as $\delta X(t)$). The other quantities are assumed to be constant in time. The static balance equations are then removed from the dynamic equations and a Fourier transformation is applied with respect to time. Assuming linear theory to be valid, the second-order terms associated with the perturbations are neglected. Using a multi-energy group approach for the discretization of the energy variable, the relationships for the evaluation of the angular neutron noise $\delta\psi_g(\vec{r},\widehat{\Omega},\omega)$ in the frequency domain, are:

$$\left[\widehat{\Omega}\cdot\nabla + \Sigma_{t,g,0}(\vec{r}) + \frac{i\omega}{v_g}\right]\delta\psi_g(\vec{r},\widehat{\Omega},\omega) = \frac{1}{4\pi}\sum_{g'}\Sigma_{s,g'\to g,0}(\vec{r})\delta\phi_{g'}(\vec{r},\omega) + \frac{1}{4\pi k}\left[\chi_{p,g}(\vec{r})\left(1-\sum_q\beta_q(\vec{r})\right) + \sum_q\chi_{q,g}(\vec{r})\frac{\lambda_q\beta_q(\vec{r})}{i\omega+\lambda_q}\right]\sum_{g'}\nu\Sigma_{f,g',0}(\vec{r})\delta\phi_{g'}(\vec{r},\omega) + S_g(\vec{r},\widehat{\Omega},\omega) \qquad (1)$$

The term $S_g(\vec{r},\widehat{\Omega},\omega)$ represents the neutron noise source and it reads as:

$$S_g(\vec{r},\widehat{\Omega},\omega) = -\delta\Sigma_{t,g}(\vec{r},\omega)\psi_{g,0}(\vec{r},\widehat{\Omega}) + \frac{1}{4\pi}\sum_{g'}\delta\Sigma_{s,g'\to g}(\vec{r},\omega)\phi_{g',0}(\vec{r}) + \frac{1}{4\pi k}\left[\chi_{p,g}(\vec{r})\left(1-\sum_q\beta_q(\vec{r})\right) + \sum_q\chi_{q,g}(\vec{r})\frac{\lambda_q\beta_q(\vec{r})}{i\omega+\lambda_q}\right]\sum_{g'}\nu\delta\Sigma_{f,g'}(\vec{r},\omega)\phi_{g',0}(\vec{r}) \qquad (2)$$

As shown in Eq. (2), the static angular neutron flux $\psi_{g,0}(\vec{r},\widehat{\Omega})$ and the static scalar neutron flux $\phi_{g',0}(\vec{r})$ are needed. Therefore, a frequency-domain neutron noise simulator has to solve the criticality problem before the neutron noise can be determined. The models thus consist of the combination of a static and a dynamic solver. The solution leads to complex quantities, from which amplitude and phase of the neutron noise are estimated.

A simulator based on Eqs. (1)-(2) is under development at Chalmers University of Technology. Such a simulator makes use of a diamond finite difference scheme for the spatial discretization and a discrete ordinates method for the angular discretization. The current version is limited to 2-dimensional geometries and treats the scattering as isotropic. Since the approach requires a heavy computational cost, the numerical acceleration of the algorithm plays an important role. A first effort in the development of the simulator was to test the Diffusion Synthetic Acceleration (DSA) method [5] for both the static and dynamic parts, with only 2 energy groups [6].

When extending the neutron noise simulator to a generic number of energy groups, the implementation of an efficient and stable accelerated scheme becomes more challenging. A typical Coarse Mesh Finite Difference (CMFD) method [7, 8] is used for the acceleration of the static solver. As starting point in the investigation of a strategy for the acceleration of the dynamic solver, the DSA method is applied to the inner iterations associated with the transport sweep within each energy group.

The DSA method for the multi-group neutron noise solver is implemented as follows. Arranging Eq. (1) in a matrix form, the multi-group, angular neutron noise column-vector $\boldsymbol{\delta\psi}^{(l,m+1/2)}$ (where m and l denote the inner and outer iteration indexes respectively) is estimated from the equation associated with the transport sweep:

$$\widehat{\Omega}\cdot\nabla\boldsymbol{\delta\psi}^{\left(l,m+\frac{1}{2}\right)}\left(\vec{r},\widehat{\Omega},\omega\right)+\boldsymbol{\Sigma}_{\mathbf{t}}^{\mathbf{dyn}}(\vec{r},\omega)\boldsymbol{\delta\psi}^{\left(l,m+\frac{1}{2}\right)}\left(\vec{r},\widehat{\Omega},\omega\right)= \frac{1}{4\pi}\boldsymbol{\Sigma}_{s,s}(\vec{r})\boldsymbol{\delta\phi}^{(l,m)}(\vec{r},\omega)+\frac{1}{4\pi}\boldsymbol{\Sigma}_{s,d}(\vec{r})\boldsymbol{\delta\phi}^{(l,M)}(\vec{r},\omega)+\frac{1}{4\pi}\boldsymbol{\Sigma}_{s,u}(\vec{r})\boldsymbol{\delta\phi}^{(l)}(\vec{r},\omega)+ \frac{1}{4\pi}\boldsymbol{\chi}(\vec{r},\omega)\boldsymbol{\nu\Sigma_{\mathbf{f}}}(\vec{r})\boldsymbol{\delta\phi}^{(l)}(\vec{r},\omega)+\mathbf{S}\left(\vec{r},\widehat{\Omega},\omega\right) \tag{3}$$

In Eq. (3), the definition of the matrices and vectors are:

$$\boldsymbol{\Sigma}_{s,s}=\begin{bmatrix}\Sigma_{s,1\to1,0} & & 0\\ & \Sigma_{s,2\to2,0} & \\ & & \cdots \\ 0 & & \Sigma_{s,G\to G,0}\end{bmatrix} \qquad \boldsymbol{\Sigma}_{s,d}=\begin{bmatrix} & \Sigma_{s,1\to2,0} & \Sigma_{s,1\to3,0} & \cdots & \Sigma_{s,1\to G,0}\\ & & \Sigma_{s,2\to3,0} & \cdots & \Sigma_{s,2\to G,0}\\ & & & \ddots & \\ & 0 & & & \Sigma_{s,G-1\to G,0}\\ & & & & \end{bmatrix}$$

$$\boldsymbol{\Sigma}_{\mathbf{t}}^{\mathbf{dyn}}=\begin{bmatrix}\Sigma_{t,1,0}+i\omega/v_1 & & 0\\ & \Sigma_{t,2,0}+i\omega/v_2 & \\ & & \ddots \\ 0 & & \Sigma_{t,G,0}+i\omega/v_G\end{bmatrix} \qquad \boldsymbol{\nu\Sigma_{\mathbf{f}}}=\frac{1}{k_{eff}}\left[\nu\Sigma_{f,1,0}\quad \nu\Sigma_{f,2,0}\quad \dots\quad \nu\Sigma_{f,G,0}\right]$$

$$\boldsymbol{\chi}=\begin{bmatrix}\chi_{p,1}\left(1-\sum_q\beta_q\right)+\sum_q\chi_{d,1,q}\frac{\lambda_q\beta_q}{i\omega+\lambda_q}\\ \chi_{p,2}\left(1-\sum_q\beta_q\right)+\sum_q\chi_{d,2,q}\frac{\lambda_q\beta_q}{i\omega+\lambda_q}\\ \vdots\\ \chi_{p,G}\left(1-\sum_q\beta_q\right)+\sum_q\chi_{d,G,q}\frac{\lambda_q\beta_q}{i\omega+\lambda_q}\end{bmatrix} \qquad \boldsymbol{\Sigma}_{s,u}=\begin{bmatrix} & & & 0\\ \Sigma_{s,2\to1,0} & & & \\ \Sigma_{s,3\to1,0} & \Sigma_{s,3\to2,0} & & \\ \vdots & & \ddots & \\ \Sigma_{s,G\to1,0} & \Sigma_{s,G\to2,0} & \cdots & \Sigma_{s,G\to G-1,0}\end{bmatrix} \tag{4}$$

The right-hand side of Eq. (3) contains five terms. The first term that represents the self-scattering source is updated from the previous m-th inner iteration. The maximum number of inner iterations for each energy group is fixed to M. The second term is the down-scattering term and it is estimated after performing the prescribed M inner iterations for the previous (higher) energy groups. The third term is the up-scattering term and the fourth term is the fission source term and they are updated after each outer iteration. The last term is a column-vector related to the neutron noise source. The angular neutron noise is integrated over the angle $\widehat{\Omega}$ to obtain the scalar neutron noise:

$$\boldsymbol{\delta\phi}^{(l,m+1/2)}=\int\boldsymbol{\delta\psi}^{(l,m+1/2)}d\widehat{\Omega} \tag{5}$$

Then the scalar neutron noise $\boldsymbol{\delta\phi}^{(l,m+1/2)}$ evaluated with Eq. (5) is introduced in the DSA equation associated with the inner iterations:

$$-\nabla \cdot \mathbf{D}(\vec{r},\omega)\nabla \mathbf{F_i}^{(l,m+1)}(\vec{r},\omega) + \mathbf{\Sigma_R}(\vec{r},\omega)\mathbf{F_i}^{(l,m+1)}(\vec{r},\omega) = \mathbf{\Sigma}_{s,s}(\vec{r})\left(\boldsymbol{\delta\phi}^{(l,m+1/2)} - \boldsymbol{\delta\phi}^{(l,m)}\right)(\vec{r},\omega) \quad (6)$$

with $\mathbf{D} = 1/\left(3\mathbf{\Sigma_t^{dyn}}\right)$ and $\mathbf{\Sigma_R} = \mathbf{\Sigma_t^{dyn}} - \mathbf{\Sigma_{s,s}}$. The components of $\mathbf{F_i}^{(l,m+1)}$ calculated from Eq. (6) for each energy group, are complex values. In a multi-group formalism, the new estimate of $\boldsymbol{\delta\phi}^{(l,m+1)}$ is obtained using only the information of the real part of $\mathbf{F_i}^{(l,m+1)}$, while the imaginary part is not modified:

$$\boldsymbol{\delta\phi}^{(l,m+1)} = Re\left(\mathbf{F_i}^{(l,m+1)} + \boldsymbol{\delta\phi}^{(l,m+1/2)}\right) + \mathrm{i} * \mathrm{Im}\left(\boldsymbol{\delta\phi}^{(l,m+1/2)}\right) \quad (7)$$

After M inner iterations, $\boldsymbol{\delta\phi}^{(l+1)}$ can be obtained:

$$\boldsymbol{\delta\phi}^{(l+1)} = \boldsymbol{\delta\phi}^{(l,\mathrm{M})} \quad (8)$$

and sent back to Eq. (3) unless the convergence criterion is met.

The solver is tested over a neutron noise problem in which a 2-D heterogeneous system is perturbed with a localized neutron noise source. The system corresponds to the C5G7 configuration [9] whose neutron cross-sections are provided in 7 energy groups for the nuclear fuel pins and the relative surrounding moderator regions separately. The neutron noise source is modelled as a fluctuation of the neutron capture cross-section in just one fuel pin. Results of this simulation together with the numerical performance of the solver will be presented at the conference. The application of the DSA method to the inner iterations is expected to have a beneficial effect. However, the acceleration of the outer iterations will be the crucial factor in the improvement of the convergence of the multi-group simulator. Future work will focus on the implementation of the CMFD method for the outer iterations in the dynamic solver of the simulator.

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