NEUTRONIC SIMULATION OF FUEL ASSEMBLY VIBRATIONS IN A NUCLEAR REACTOR

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ABSTRACT

The mechanical vibrations of fuel assemblies have been shown to give rise to high levels of neutron noise, triggering in some circumstances the necessity to operate nuclear reactors at a reduced power level. This work simulates and analyses the effect in the neutron field of the oscillation of one single fuel assembly without considering thermal-hydraulic feedback. The amplitude of the fuel assembly vibration ranges from 0 to 1 millimetres and this implies the use of fine meshes and accurate numerical solvers. Results show two different effects in the neutron field caused by the fuel assembly vibration. First, a global slow variation of the total neutron power due to a change in the criticality of the system. Second, an in-phase change in the neutron flux with the assembly vibration. This second effect is more important and has a strong spatial dependence. This paper shows a comparison between a time domain analysis and a frequency domain analysis of the phenomena, in order to validate the time domain solution against the frequency domain solution. Numerical results shows a really close match between these two approaches.

KEYWORDS: Neutron noise, fuel assembly vibrations, neutron diffusion, frequency domain, time domain

1. INTRODUCTION

Being able to monitor the state of nuclear reactors while they are running at nominal conditions is a safety requirement. The early detection of anomalies gives the possibility to take proper actions before such problems lead to safety concerns or impact plant availability. The CORTEX project [1], funded by the European Commission in the Euratom 2016-2017 work program, aims at developing an innovative core monitoring technique that allows detecting anomalies in nuclear reactors, such as excessive vibrations of core internals, flow blockage, coolant inlet perturbations... The technique is based on using the inherent fluctuations in neutron flux recorded by in-core and ex-core instrumentation, referred as *neutron noise*, from which the anomalies will be detected.

The vibration of fuel assemblies is suspected to be a possible cause of increased neutron noise due to the correlation existing between the stiffness of the installed fuel assemblies (FA) and the noise

amplitudes [2]. The objective of this paper is twofold. First, the assessment of the capability of time and frequency domain codes to simulate the propagation in the neutron field of FA vibrations. Second, to evaluate the order of magnitude and spatial shape of the neutron noise behaviour when a single FA is vibrating. This contribution investigates the influence of lateral deflection of one FA by moving the whole homogeneous material back and forth following a sinusoidal oscillation.

Other studies considered FA vibrations by randomly modifying the size of the water gaps which surrounds the FA of interest [3]. Also, the influence of coherent lateral deflections for all FAs on the neutron flux have been considered in [2] and [4].

Mechanical vibrations of FA have been studied from a structural point of view in [5] and [6]. Natural frequencies reported in literature range from 0.8 Hz to 24.5 Hz depending on the mode and the idealized form of bearing. The amplitude of the vibrations ranges can reach up to 1 mm. Fuel elements can be mechanically described in a first approach by a damped spring with hysteresis. There are possibilities for amplitudes larger than 1 mm in case of hypothetical synchronous motions and forced excitations. The usual FA vibration model cover motions in the parameter field from 0.5 to 25 Hz and amplitudes up to 5 mm in a sinusoidal movement. This work is based on FA vibration of 1 Hz and 1 mm of amplitude as a usual example.

This paper is organized as follows. First, the time domain analysis of the effect of the FA vibration is explained in Section 2. Then, Section 3 describes briefly the frequency domain analysis and how the perturbation is included in this context. After that, some numerical results are given in Section 4 to verify the proposed methodologies. Finally, Section 5 summarizes the main conclusions of the study.

2. TIME DOMAIN ANALYSIS

The neutron transport equation is usually approximated by the neutron diffusion equation. For a given transient, the balance of neutrons inside a nuclear reactor core can be modelled using the time dependent neutron diffusion equation in the two energy groups approximation without up-scattering. This model is of the form of

$$[\mathrm{v}^{-1}]\frac{\partial \Phi}{\partial t} + \mathcal{L}\Phi = (1-\beta_{eff})\mathcal{M}\Phi + \sum_{k=1}^{K}\lambda_k \chi \mathcal{C}_k, \tag{1}$$

$$\frac{\partial \mathcal{C}_k}{\partial t} = \beta_k[\nu\Sigma_{f1}\ \nu\Sigma_{f2}]\Phi - \lambda_k \mathcal{C}_k, \qquad k = 1, \ldots, K, \tag{2}$$

where K is the number of delayed neutron precursors groups considered and the matrices are defined as

$$\mathcal{L} = \begin{pmatrix} -\vec{\nabla}\cdot(D_1\vec{\nabla}) + \Sigma_{a1} + \Sigma_{12} & 0 \\ -\Sigma_{12} & -\vec{\nabla}\cdot(D_2\vec{\nabla}) + \Sigma_{a2} \end{pmatrix}, \quad [\mathrm{v}^{-1}] = \begin{pmatrix} \frac{1}{\mathrm{v}_1} & 0 \\ 0 & \frac{1}{\mathrm{v}_2} \end{pmatrix},$$

$$\mathcal{M} = \begin{pmatrix} \nu\Sigma_{f1} & \nu\Sigma_{f2} \\ 0 & 0 \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

where ϕ_1 and ϕ_2 are the fast and the thermal neutron fluxes, respectively. Other quantities have their usual notation in the nuclear reactor engineering context.

In this work, a high order finite element method (FEM) has been used for the discretization of the neutron diffusion equation, as the one presented in [7] and [8]. The code developed is called *FEMFFUSION*.

Once the spatial discretization has been selected, a discrete version of the time dependent neutron diffusion equation is solved. Since the system of ordinary differential equations resulting from the discretization of the neutron diffusion equations is, in general, stiff, implicit methods are necessary. Particularly, a first order backward method is used [9]. This method leads to a large system of linear equations that needs to be solved for each time step. The semi-discrete two energy groups time dependent neutron diffusion equation together with the neutron precursors concentration equations are of the form

$$[\tilde{\mathrm{v}}^{-1}]\frac{d\Phi}{dt} + L\Phi = (1-\beta)M\Phi + \sum_{k=1}^{K}\lambda_k X C_k \,, \tag{3}$$

$$P\frac{dC_k}{dt} = \beta_k \left(M_{11}M_{21}\right)\Phi - \lambda_k P C_k \,, \quad k = 1, \ldots, K \,, \tag{4}$$

where L and M are the matrices obtained from the spatial discretization of operators $\mathcal{L}$ and $\mathcal{M}.\Phi$ and C_k are the vector of coefficients of the neutron flux and the precursors concentration in terms of the polynomials used in the FEM. The matrices X and $[\tilde{\mathrm{v}}^{-1}]$ are defined as

$$X = \begin{pmatrix} P \\ 0 \end{pmatrix} \,, \qquad [\tilde{\mathrm{v}}^{-1}] = \begin{pmatrix} P\,\mathrm{v}_1^{-1} & 0 \\ 0 & P\,\mathrm{v}_2^{-1} \end{pmatrix} \,,$$

where the matrix P is the mass matrix of the spatial discretization, which appears due to the fact that the polynomial basis used in the spatial discretization is not orthogonal.

The neutron precursors equation (4) can be discretized in time using an explicit scheme as

$$PC_k^{n+1} = PC_k^n e^{-\lambda_k \Delta t} + \frac{\beta_k}{\lambda_k}\left(1 - e^{-\lambda_k \Delta t}\right)\left(M_{11}^{n+1}M_{12}^{n+1}\right)\,\Phi^{n+1} \,. \tag{5}$$

In the same way, Euler's backward method is used in equation (3) and taking into account equation (5), we get a system of linear equations,

$$T^{n+1}\Phi^{n+1} = R^n\Phi^n + \sum_{k=1}^{K}\lambda_k e^{-\lambda_k \Delta t} X C_k^n \,, \tag{6}$$

where the matrices are defined as,

$$T^{n+1} = \frac{1}{\Delta t}[\tilde{\mathrm{v}}^{-1}] + L^{n+1} - \hat{a}M^{n+1} \,,$$
$$R^n = \frac{1}{\Delta t}[\tilde{\mathrm{v}}^{-1}] = \frac{1}{\Delta t}\begin{pmatrix} P\,\mathrm{v}_1^{-1} & 0 \\ 0 & P\,\mathrm{v}_2^{-1} \end{pmatrix} \,,$$

and the coefficient $\hat{a}$ is

$$\hat{a} = 1 - \beta_{eff} + \sum_{k=1}^{K}\beta_k\left(1 - e^{-\lambda_k \Delta t}\right) \,.$$

This system of equations has to be solved for each time step, $\Delta t = 10^{-4}$. The preconditioned BICGSTAB method [10] has been chosen to solve these systems and the preconditioner used has been the incomplete LU preconditioner. This system of equations is large and sparse and has to be solved for each new time step with a high accuracy, tol =10^{-12}. Large CPU times are required to simulate a full transient.

Finally, to compare the time domain computations with frequency domain analysis and to evaluate the neutron noise, we must define the flux noise as

$$\delta\phi(\mathbf{r},\, t) = \phi(\mathbf{r},\, t) - \phi(\mathbf{r},\, 0). \tag{7}$$

Then, we can apply a Fourier transform to the flux noise in the time domain,

$$\delta\phi(\mathbf{r},\, \omega) = \mathcal{F}[\delta\phi(\mathbf{r},\, t)] = \int_{-\infty}^{\infty} \exp(-i\omega t)\delta\phi(\mathbf{r},\, \omega)\mathrm{d}t. \tag{8}$$

This transform is applied numerically using the Fast Fourier Transform (FFT) algorithm over the time dependent results. In the case of FA vibrations, the frequency space can be cut for the frequency of the assembly oscillation because this is the predominant frequency in the noise spectrum, as it is shown in the next Section.

3. FREQUENCY DOMAIN ANALYSIS

The *CORE SIM* tool [11] solves the neutron diffusion equation in the frequency domain and has been already successfully used to study other neutron noise sources, [12], [13], [14], [15]. This tool uses a central difference method to solve the two-group diffusion equation in the frequency domain for the first-order neutron noise approximation. The tool, specifically made for noise calculations, has also been verified against many (semi-)analytical solutions [16].

It must be taken into account that *FEMFFUSION* allows any number of delayed neutron precursors groups while *CORE SIM* only uses 1 group of precursors. In this work, the usual definition of 6 groups of precursors is used in the time domain calculations. For the *CORE SIM* calculation, 1 group effective vales, β_{eff} and λ_{eff}, are used. However, as it is shown in this work, this approximation does not have any appreciable effect on the accuracy of the results for usual natural FA vibration frequencies.

The first-order neutron noise theory is based on splitting every time dependent term, expressed as $U(\mathbf{r}, t)$, into their mean value, U_0, and their fluctuation around their mean value, δU_0 as

$$U(\mathbf{r},\, t) = U_0(\mathbf{r}) + \delta U(\mathbf{r},\, t). \tag{9}$$

The fluctuations are assumed to be small compared to the mean values. This allows to neglect second-order term $(\delta U(\mathbf{r},\, t) \times \delta U(\mathbf{r},\, t)) = 0$. Also, the fluctuations of the diffusion coefficients are neglected and $\delta D_g = 0$ is assumed . Thus, the first-order neutron noise equation can be written as

$$\left(\vec{\nabla} D \vec{\nabla} + \Sigma_{\text{dyn}}\right) \begin{pmatrix} \delta\phi_1 \\ \delta\phi_2 \end{pmatrix} = \delta\Sigma_r \phi_r + \phi_a \begin{pmatrix} \delta\Sigma_{a1} \\ \delta\Sigma_{a2} \end{pmatrix} + \phi_f \begin{pmatrix} \delta\Sigma_{f1} \\ \delta\Sigma_{f2} \end{pmatrix}, \tag{10}$$

where

$$D = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix}, \qquad \Sigma_{\text{dyn}} = \begin{pmatrix} -\Sigma_1 & \nu\Sigma_{f2}\left(1 - \frac{i\omega\beta_{eff}}{i\omega+\lambda_{eff}}\right) \\ \Sigma_{12} & \left(\Sigma_{a2} + \frac{i\omega}{\mathrm{v}_2}\right) \end{pmatrix}, \qquad \phi_r = \begin{pmatrix} \phi_1 \\ -\phi_1 \end{pmatrix},$$

$$\phi_a = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix}, \qquad \phi_f = \begin{pmatrix} -\phi_1\left(1 - \frac{i\omega\beta_{eff}}{i\omega+\lambda_{eff}}\right) & -\phi_2\left(1 - \frac{i\omega\beta_{eff}}{i\omega+\lambda_{eff}}\right) \\ 0 & 0 \end{pmatrix},$$

$$\Sigma_1 = \Sigma_a + \frac{i\omega}{\mathrm{v}_1} + \Sigma_r - \nu\Sigma_{f1}\left(1 - \frac{i\omega\beta_{eff}}{i\omega + \lambda_{eff}}\right).$$

The neutron noise equation is an inhomogeneous equation with complex quantities that has to be solved after the static solution is obtained, as ϕ_1 and ϕ_2 represent the steady state fast and thermal neutron fluxes. The related static eigenvalue problem must be solved with the same spatial discretization as the frequency domain neutron noise equation to get coherent results.

3.1. Modelling a vibrating fuel assembly in the frequency domain

Now, we describe how to model the vibration of a FA as a perturbation of cross sections in the frequency domain. One assumes that the cross section, Σ_α, at the interface $x = b$ between two material regions, as the one shown in Figure 1 is described as:

$$\Sigma_\alpha(x) = (1 - \mathcal{H}(x - b))\,\Sigma_\alpha^I + \mathcal{H}(x - b)\Sigma_\alpha^{II}, \tag{11}$$

where $\mathcal{H}$ is the unit step function, Σ_α^I and Σ_α^{II} are the cross sections at region I and II, respectively. Therefore, a vibrating assembly can be described as two in-phase moving interfaces (one moving interface on one side of the assembly and another moving interface on the other side of the moving assembly). For the sake of simplicity, only one moving interface is first considered in the derivations presented hereafter.

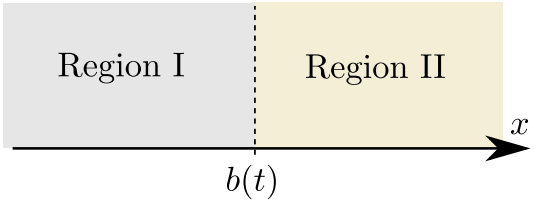


Figure 1: Vibrating interface between two regions.

An interface moving as $b(t) = b_0 + A\sin(\omega_p t)$, results in:

$$\Sigma_\alpha(x,\, t) = (1 - \mathcal{H}(x - b_0 - A\sin(\omega_p t)))\,\Sigma_\alpha^I + \mathcal{H}(x - b_0 - A\sin(\omega_p t))\Sigma_\alpha^{II}. \tag{12}$$

Using the first order Taylor expansion, the cross section perturbation can be expressed as

$$\delta\Sigma_\alpha(x,t) = \left(\Sigma_\alpha^I - \Sigma_\alpha^{II}\right) \times A\sin(\omega_p t)) \times \delta(x - b_0), \tag{13}$$

and in the frequency domain the perturbation is written as follows

$$\delta\Sigma_\alpha(x,\omega) = \frac{1}{2} i \left(\Sigma_\alpha^I - \Sigma_\alpha^{II}\right) \delta(x - b_0)\delta(\omega - \omega_p). \tag{14}$$

Since in numerical tools like *CORE SIM* the perturbation is introduced node-wise, one could assume that the perturbed region is $x \in [b_0 - A,\ b_0 + A]$ with a perturbation value of $\delta\Sigma_\alpha = \frac{1}{2} i \left(\Sigma_\alpha^I - \Sigma_\alpha^{II}\right)$. If the mesh used does not match the perturbed region, the perturbation must be rescaled.

To ensure the validity of the first order approximation, a numerical fast Fourier transform of the time dependent cross sections is calculated. The numerical FFT was obtained for a perturbation of 1 Hz. Figure 2 shows the spectrum of the perturbation at $x = b_0$ using the FFT. The amplitude of the cross section perturbation is maximum at 1 Hz, ω_p, and the other frequencies are at least three times smaller while the first order approximation only uses the perturbation at 1 Hz. Figure 3 shows the amplitude of the perturbation $|\delta\Sigma_\alpha|$ at one 1 Hz for the FFT and the first order approximation. It can be seen that the first order approximation has the same perturbation integral as the FFT and it can be introduced using only one node.

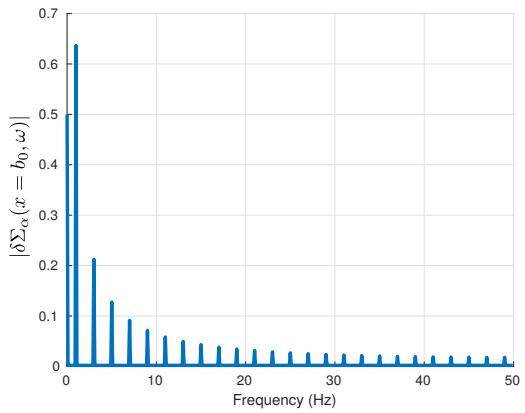


Figure 2: Spectrum of the numerical FFT at b_0.

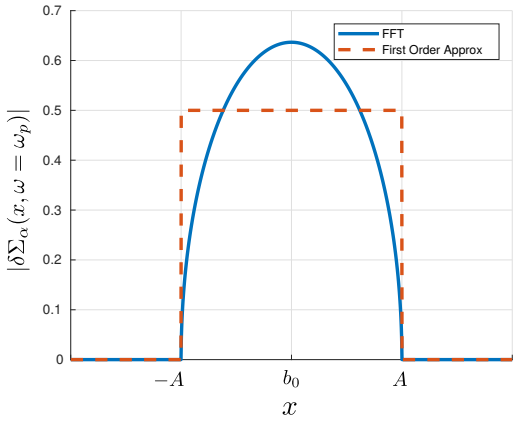


Figure 3: Perturbation amplitude at ω_p.

4. NUMERICAL RESULTS

The BIBLIS 2D benchmark is selected to verify the frequency and time domain analysis for a vibrating FA. This is a classical two-group neutron diffusion problem taken as a benchmark for different numerical codes. The definition of the benchmark can be found in [17]. The problem is made critical before starting the time dependent calculation by dividing $\nu\Sigma_{fg}$ by the previously calculated multiplicative factor of the reactor, k_{eff}.

The assembly in the position $(6,\ 6)$ is selected to be oscillating along the x direction as

$$x_i(t) = x_{i0} + A\sin(\omega_p t), \tag{15}$$

where $x_i(t)$ represents each position of the vibrating assembly along time, originally placed at x_{i0}. Due to the different scales of the problem, a fine mesh needs to be used to accurately solve the system. In the time-domain analysis, a refined mesh in the surroundings of the moving FA with 869 cells and cubic polynomials in the FEM is used. In the frequency domain analysis, a uniform mesh of 4624 cells is employed. If these fine meshes are not used, the effect of the FA vibration could be overestimated.

Figure 4a shows the total power evolution for an oscillation of 1 mm of amplitude and a frequency of 1 Hz along 3 periods. It must be noted that the total neutron power is normalized to 1.0 for unperturbed operation conditions. A sinusoidal change in the total power with a really small amplitude, about 3.8e-7, with a linear increment over time is observed. This increment is caused by the change of the criticality of the system when the assembly is moved from its starting position. Figure 4b displays the change of the static multiplicative factor of the reactor, δk_{eff}, through the positions travelled by the moving FA during one period. It can be seen that the change in the k_{eff} is about 6.0e-8 in the amplitude and the integral is positive during one period. The behaviour of the total power is analogous with the one studied analytically in a point kinetic reactor model in [18] and [19] where the linear increase of neutron power is caused by delayed neutrons. However, thermal-hydraulic feedback will mitigate the slow increment in the total power.

Figure 5 displays the spatial distribution of the amplitude of the neutron noise in the frequency domain calculated with the time dependent code *FEMFFUSION*. It can be seen that the induced fast neutron noise has an influence on larger scales through the reactor compared to the thermal noise, due to the larger mean free path of fast neutrons. Otherwise, the thermal noise is localised in the surroundings of the oscillating FA. Two clear peaks can be observed in these Figures. They correspond to the perturbed region where the cross sections change along the FA movement. Figure 6 shows a comparison of the amplitude of neutron flux noise for the fast and thermal groups along $y = 150.2969$ cm between the *FEMFFUSION* code and the *CORE SIM* code, in other words, between the time-domain and the frequency-domain approaches. Figure 7 gives a comparison of the phase of the neutron noise between the two codes. A really close agreement is observed for both the amplitude and the phase of the neutron noise validating the time-domain analysis against the frequency-domain one.

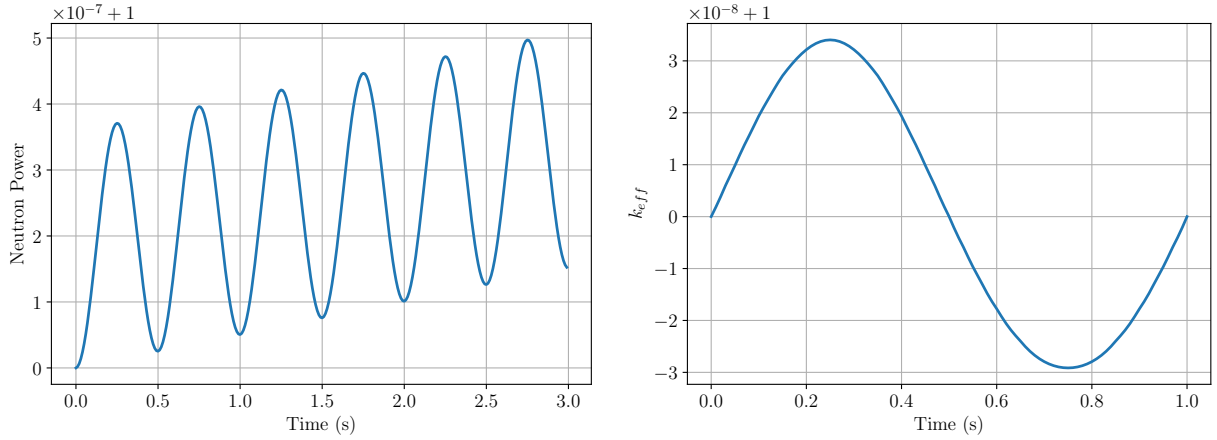
5. CONCLUSIONS

The current research is an attempt to understand in more detail the coupling mechanism between the mechanical vibration of fuel assemblies and the generated neutron noise. The problem combines different spatial scales, e.g. a 1 mm oscillation of a fuel assembly that measures about 20 cm in a reactor of some meters size. This implies that we need to work with a very high precision in the spatial discretization and in the tolerances given to the solvers.

Numerical results show two different effects in the neutron field caused by the fuel assembly vibration. First, a global slow variation of the total neutron power due to a change in the criticality of the system is observed. This effect is really small and will be compensated by the thermal-hydraulic coupling. Secondly, an oscillation in the neutron flux at the same frequency as the one of the assembly vibration is demonstrated. The corresponding neutron noise is highly spatially dependent. For this second effect, this paper shows a comparison between a time domain analysis and a frequency domain analysis of the phenomena. Numerical results shows a really close

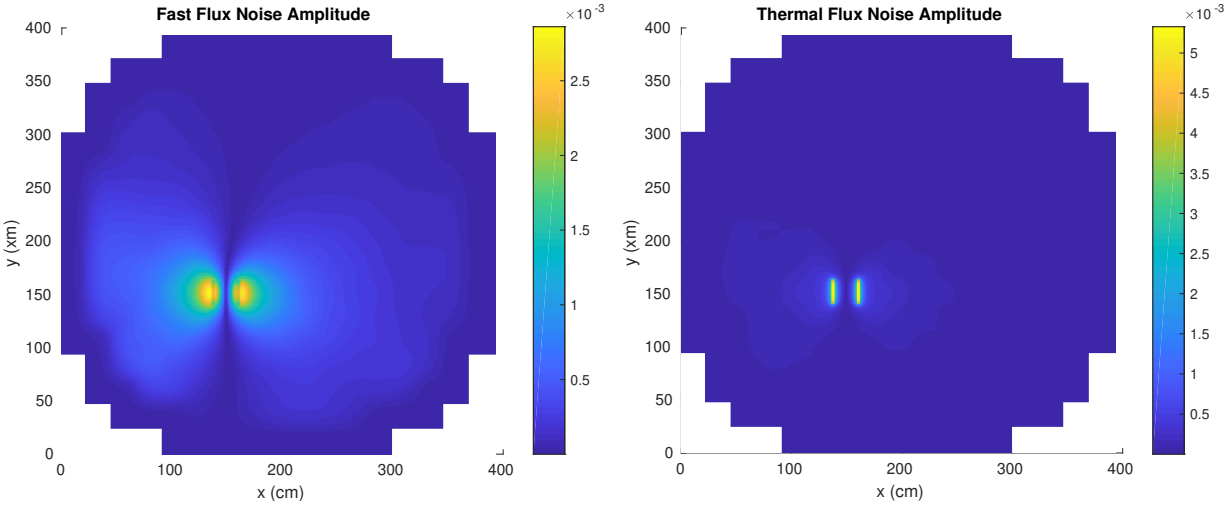
match between these two approaches, thus validating the time-domain modelling. This paper also demonstrates that neglecting second order effects, as done in the frequency-domain simulations, gives essentially the same results as not neglecting those, as done in the time-domain simulations. Also, it must be noted that the frequency domain analysis takes much less computational time than the full time domain analysis.

Future works will aim to study three dimensional cases where the FA has different axial shapes of the vibration depending on their mechanical parameters.



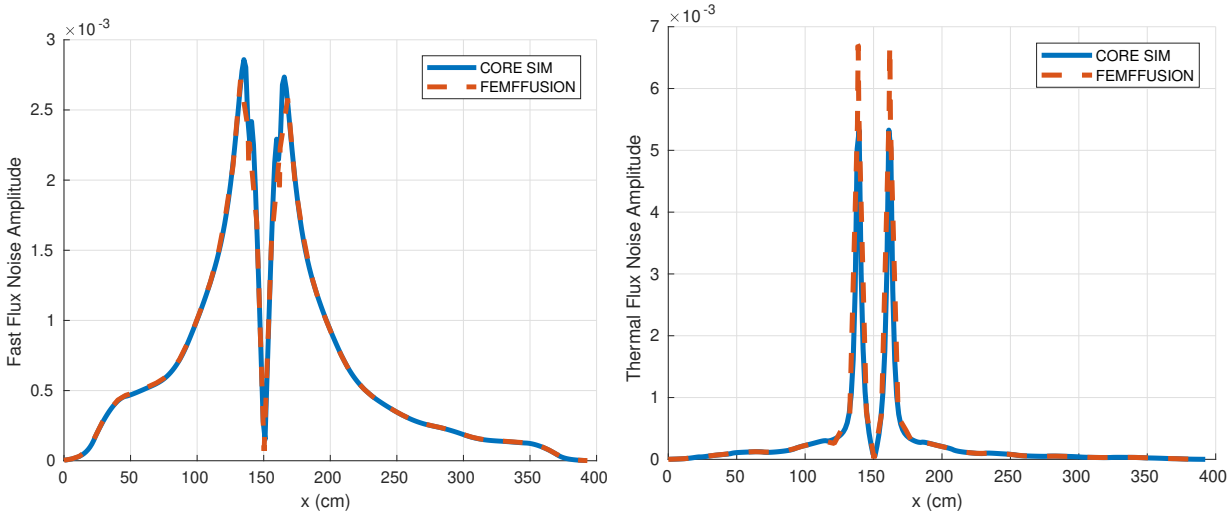
(a) Power Evolution **(b) Multiplicative factor**

Figure 4: Global results for the 2D BIBLIS where the assembly in position (6, 6) is vibrating.



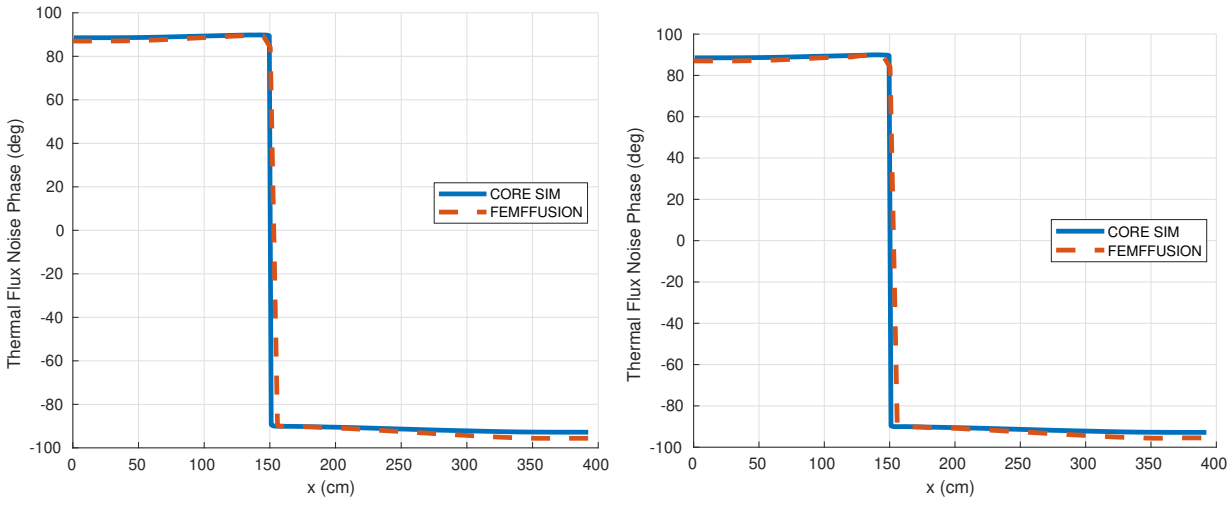
(a) Fast flux noise **(b) Thermal flux noise**

Figure 5: Noise amplitudes for the 2D BIBLIS reactor.



(a) Fast flux noise **(b) Thermal flux noise**

Figure 6: Noise amplitude comparison for 2D BIBLIS reactor in $y = 150.2969$ cm.



(a) Fast group **(b) Thermal group**

Figure 7: Phase comparison for 2D BIBLIS reactor in $y = 150.2969$ cm.

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